Distributed Generation of Mobile Call Graphs with DPLN Degree Distribution

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Outline

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Motivation

Why do we need random graphs?

- One needs to test network analysis algorithms on huge graphs
- Modern networks reach several hundred million nodes
- Real data gathering requires huge time and resource costs; lack of datasets

Mobile Call Graphs (MCG):

- **nodes** — mobile cell users
- **edges** — interaction between users (calls, SMS, etc)
- **edge weight** — number of calls or its duration

Properties of MCG:

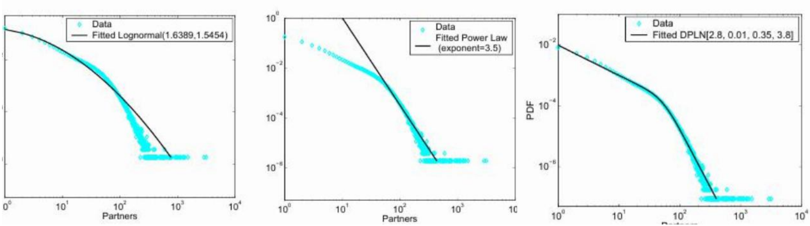
- **Degree distribution**
  - either Power Law distribution: $P(d) \sim d^{-\beta}$
  - or Double Pareto LogNormal distribution: $P(d) \sim DPLN(d, \alpha, \beta, \nu, \tau)$
- **Weights**
  - Good correlation between call duration and its amount
  - Distributed according to DPLN
- **Clusters and Clustering**
  - Heavy-tailed membership and community size distribution
  - High clustering coefficient

- Pareto: $f(x) = \begin{cases} \frac{\beta\, x_m^\beta}{x^{\beta+1}} & x \geq x_m \\ 0 & x < x_m \end{cases}$
- LogNormal:
  $X \sim LogN(\mu, \sigma^2) \Leftrightarrow \ln X \sim \mathrm{N}(\mu, \sigma^2)$
- **Double Pareto LogNormal**:

$$f(x, \alpha, \beta, \nu, \tau) = \frac{\alpha\beta}{\alpha+\beta}\left[e^{\alpha\nu+\alpha^2\tau^2/2}x^{-\alpha-1} \times \right.$$
$$\left. \times\, \mathrm{F}\left(\frac{\log x - \nu - \alpha\tau^2}{\tau}\right) + x^{\beta-1}e^{-\beta\tau+\beta^2\tau^2/2}\,\mathrm{F}^c\left(\frac{\log x - \nu + \beta\tau^2}{\tau}\right)\right],$$

where $F(x)$ - cdf of $\mathcal{N}(0,1)$, $F^c(x) = 1 - F(x)$

DPLN-distribution in pictures[1]:



[1] M. Seshadr et al. „Mobile call graphs: beyond power-law and lognormal distribution“

Problem Statement

Devise a random graph generator which allows to generate:

- networks fulfilling main properties of the mobile networks (DPLN degree distribution, community structure, etc);
- large graphs in a reasonable time.

Generators:

- GEDIS Studio Call Detail Record Generator[2]
- cdr-gen[3]
- Synthetic Call Detail Record generator using Spark[4]

[2] http://www.gedis-studio.com/online-call-detail-records-cdr-generator.html

[3] https://github.com/mayconbordin/cdr-gen

[4] https://github.com/RealImpactAnalytics/cdr-generator

Random MCG models:

- Migration model[5]
- Treasure-Hunt[6]
- Lognormal multiplicative process[7]

Real datasets:

- Wu et al. dataset (SMS)[8]
- VAST 2008 (cell interactions)[9]

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[5] Lambiotte et al. „Geographical dispersal of mobile communication networks“

[6] Nanavati et al. „On the structural properties of massive telecom call graphs: findings and implications“

[7] Seshadri et al. „Mobile call graphs: beyond power-law and lognormal distributions“

[8] http://www.pnas.org/content/107/44/18803.abstract

[9] https://www.cs.umd.edu/hcil/VASTchallenge08

Main steps:

- Two heavy-tailed sequences generation (membership and community size distributions)
- Communities generation
- Edges generation

Proposed method

Phase 1: heavy-tailed sequences generation

Parameters:

- $\alpha, \beta, \gamma, \nu$
- x_{min}, x_{max}, y_{min}, y_{max}
- N

- Need to find C such that: $\int_{x_{min}}^{x_{max}} C \cdot f(x)dx = 1$. In discrete case: $C = \left(\sum_{x_{min}}^{x_{max}} f(x)\right)^{-1}$
- $\mathbb{E}(X) = \sum_{x} Cf(x) \cdot x$
- $N_1 \cdot \mathbb{E}(X) = N_2 \cdot \mathbb{E}(Y)$
- for each $x \in [x_{min}, x_{max}]$ add to sequence list $Cf(x) \cdot N$ elements with value x:
- $\underbrace{[10, 9, 9, \ldots, 1, 1, 1]}_{N}$

- d_i^1 — number of communities i^{th} user belongs to.
  d_j^2 — size of j^{th} community.
- from each sequence $[d_1, d_2, d_3, \dots]$ we create $[d_1, d_1 + d_2, d_1 + d_2 + d_3, \dots]$

- $D_k^1 = \sum_{i=1}^{k} d_i^1 \qquad D_k^2 = \sum_{j=1}^{k} d_j^2$
- $M = N_1 \cdot D_{N_1}^1$
- $x \sim u[1, D_{N_1}^1]$
  $y \sim u[1, D_{N_2}^2]$
- $p : x \in (d_p^1, d_{p+1}^1]$
  $q : y \in (d_q^2, d_{q+1}^2]$
- (p, q) — user p belongs to community q

Parameters:

- α, β, γ
- $\delta_{in}, \delta_{out}$

To choose node v according to $d + \delta$ ($v \sim d(v) + \delta$) in graph $G(t)$ means: $P(v = v_i) = \frac{d(v_i)+\delta}{t+\delta \cdot n(t)}$, $d(v_i)$ is a degree of v_i and $n(t)$ is a number of nodes at the time t.

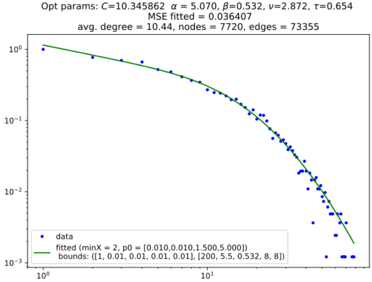
- G_0 is a graph without edges with a single node; $t_0 = 0$
- At the time stemp t graph $G(t+1)$ is created:
  1. $P = \alpha$: $w \sim d_{in} + \delta_{in}$
     new v; add $(v \to w)$
  2. $P = \beta$: $w \sim d_{in} + \delta_{in}$
     $v \sim d_{out} + \delta_{out}$
     add $(v \to w)$
  3. $P = \gamma$: $v \sim d_{out} + \delta_{out}$
     new w; add $(v \to w)$

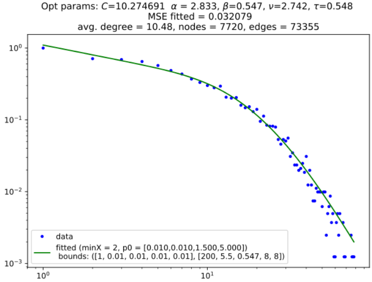
Weights of generated edges could be:

- generated randomly from Power Law distribution;
- equal to number of common membership of two nodes;
- unspecified (unweighted case).

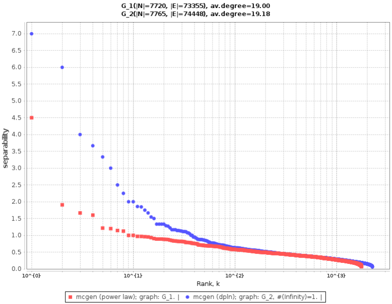
Accuracy Evaluation

Degree distribution

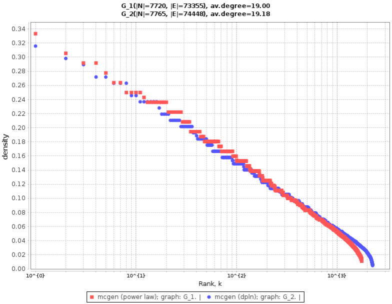




Separability



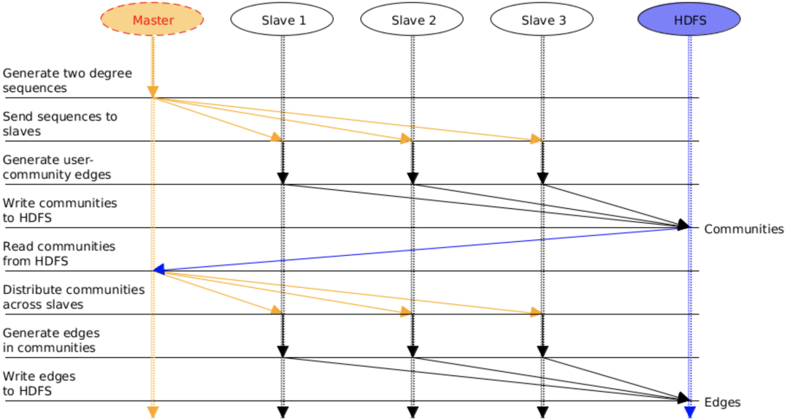
Density



$$g(S) = m_s / c_s \qquad f(S) = \frac{m_s}{|S|(|S|-1)}$$

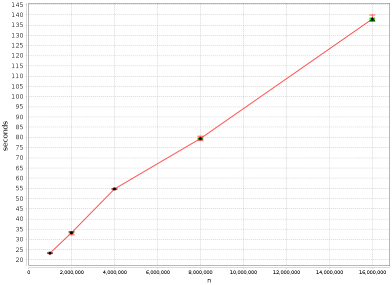
$$m_s = \sum_{i,j \in S} w_{ij} \qquad c_S = \sum_{i \in S, j \in \overline{S}} w_{ij}$$

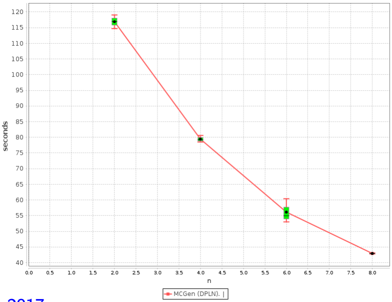
Performance Evaluation



Performance Evaluation

- For scalability by number of nodes (left) we used 4 `Spark.2xlarge` instances (64Gb, 8 CPUs) nodes.
- For scalability by number of slaves (right) we generated graph with $N = 8 \cdot 10^6$ on 4 `Spark.xlarge` instances (32Gb, 4 CPUS).





- Introduced new method for mobile call graph generation
- Method shows near-linear scalability both on the number of slaves and on the number of nodes
- Extended community statistics for weighted and directed case

Questions?
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