Numerical simulation of MHD control for high-speed non-equilibrium flow

Ioffe Physical Technical Institute

Сектор Численного Моделирования

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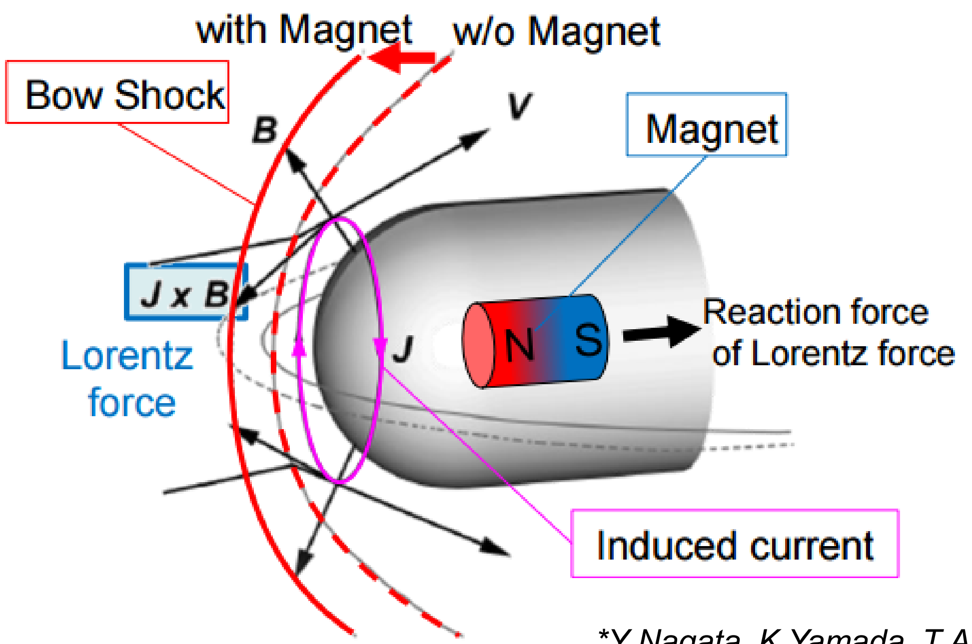
MHD flow control concept

Magnetic field interacts with weakly ionized plasma between the bow shock and the body:

- *Lorentz force*
- *Joule heating*

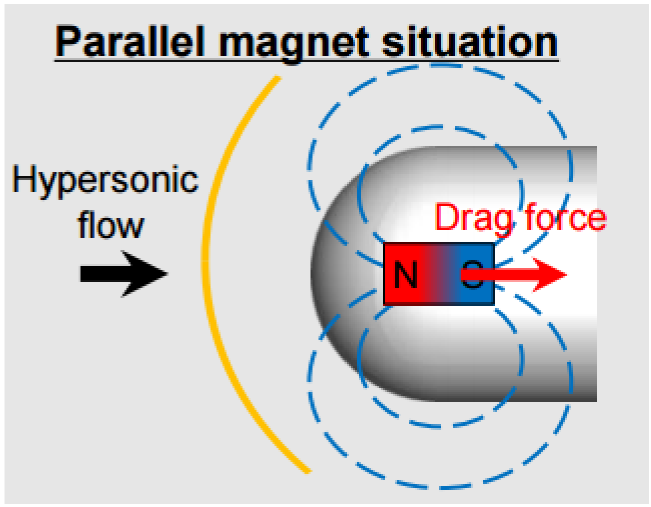
Applications:

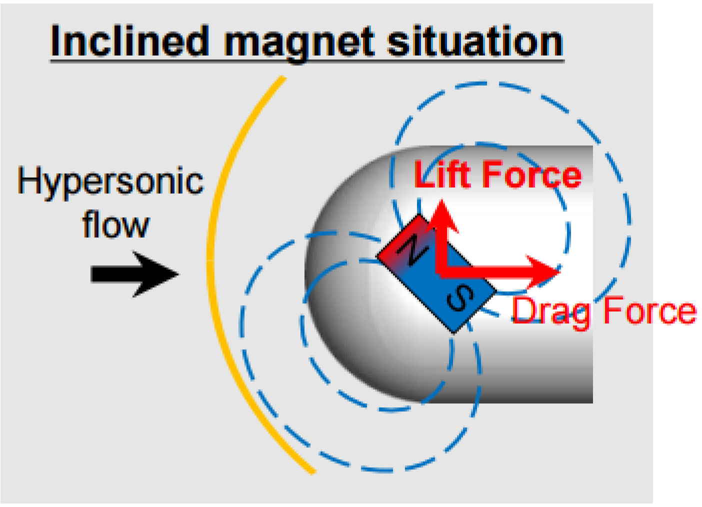
- *Aerodynamic heating reduction*
- *Aerobraking*
- *Scramjet engine inlet optimization*
- *Communication blackout mitigation*
- *Wave drag cancellation*



*Y Nagata, K Yamada, T Abe

Controlling reentry flight trajectory





Modern MHD flow control research

	Bityurin, Bocharov	Bisek, Boyd	Fujino, Ishikawa	Joussot, Coumar(*)	Guelhan	Borghi, Neretti(**)
Country of origin	Russia	USA	Japan	France	Germany	Italy
Affiliation	JIHT RAS	University of Michigan	Institute of Electrical Engineers of Japan	CNRS	DLR	University of Bologna
				European Space Agency		

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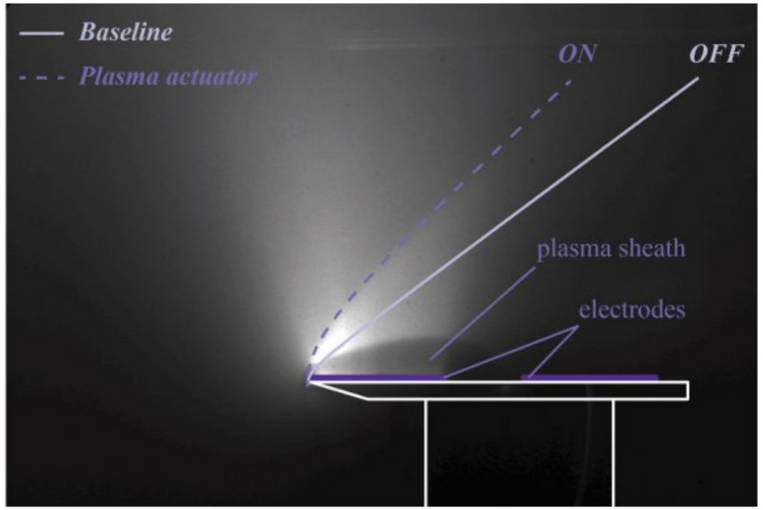

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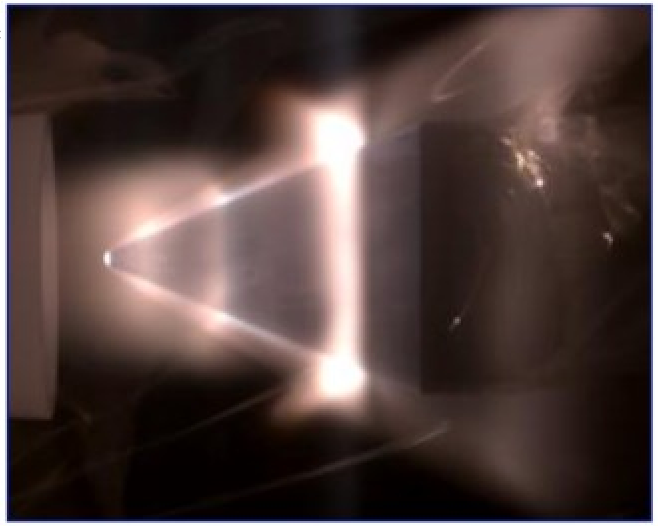


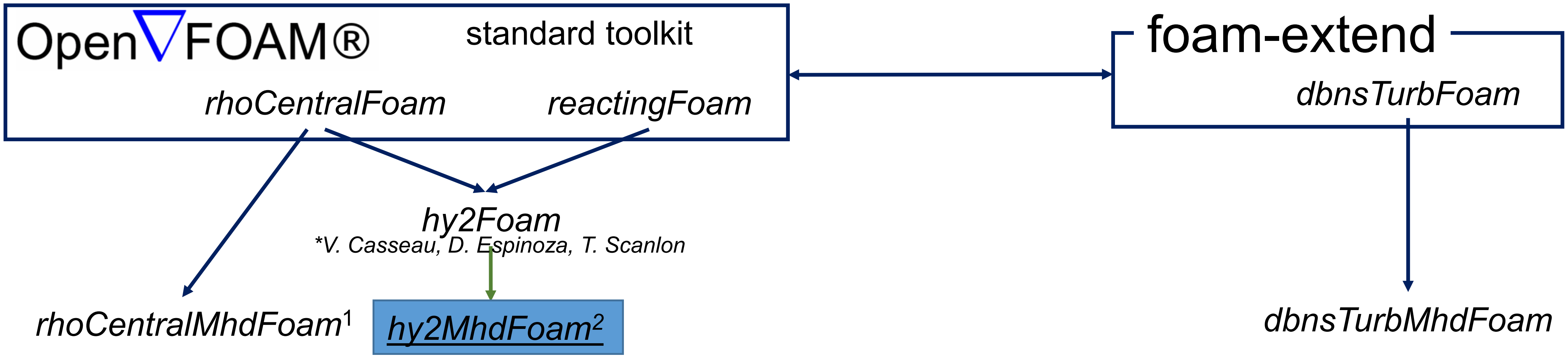
Research goals

Hypersonic flow codes

- NASA's DPLR (Data-Parallel Line Relaxation)
- LAURA (Langley Aerothermodynamic Upwind Relaxation Algorithm)
- VULCAN (Viscous Upwind aLgorithm for Complex flow Analysis)
- LeMANS (The Michigan Aerothermodynamic Navier-Stokes solver) **(has MHD capabilities)**
- US3D (UnStructured 3Dm University of Minnesota)

Solver development	MHD flow control research
• Develop an open-source solver for hypersonic non-equilibrium flows interacting with applied magnetic field • Implement different electric conductivity models to test their applicability for the MHD flow control simulation • Implement the models for associated phenomena, such as Hall effect, Ion slip and radiative heat transfer	• Studying the interaction between high-speed ionized flow and magnetic field • Investigating feasibility and the potential of MHD flow control technology

OpenFoam solver development



rhoCentralFoam: Kurganov-Tadmor central difference schemes
dbnsTurbFoam: Godunov-type HLLC-Roe scheme

[1]Ryakhovskiy, A. I., & Schmidt, A. A. (2016). MHD supersonic flow control: OpenFOAM simulation. *Труды института системного программирования РАН*, *28*(1).
[2]Ryalhovskiy, A. I., Schmidt, A. A., Antonov, V. I. (2017) Numerical Simulation of MHD Hypersonic Flow Control, *Proceeding of International Symposium on Magneto-plasma Aerodynamics, Ed. V. Bityurin,* JIHT

hy2Foam/hy2MhdFoam features

hy2Foam	
rhoCentralFoam Density-based solver for supersonic flow	***reactingFoam*** Solver for subsonic combustion
Kurganov-Tadmor schemes	**Chemical reactions mechanism (Arrhenius)**
Two-temperature approach with chemistry-vibrational coupling • Park TTv two-temperature model • CVDV model	
Multispecies diffusion • Wilk • Gupta • Armally-Sutton	
hy2MhdFoam	
MHD terms for Navier-Stokes equation • Lorentz force • Joule heating	**Electrical conductivity models** • Boltzmann • Spitzer-Harm • Chapman-Cowling • Bush • Raizer
Radiative heat transfer models • P1	

hy2Foam+MHD

$$\frac{\partial \boldsymbol{\mathcal{U}}}{\partial t} + \frac{\partial\left(\boldsymbol{\mathcal{F}}_{i,inv} - \boldsymbol{\mathcal{F}}_{i,vis}\right)}{\partial x_i} = \dot{\boldsymbol{\mathcal{W}}}.$$

$$\boldsymbol{\mathcal{U}} = \begin{pmatrix} \rho \\ \rho_s \\ \rho u \\ \rho v \\ \rho w \\ \varepsilon_{ve,m} \\ \varepsilon \end{pmatrix} \quad \boldsymbol{\mathcal{F}}_{i,inv} = \begin{pmatrix} \rho u_i \\ \rho_s u_i \\ \rho u_i u + \delta_{i1} p \\ \rho u_i v + \delta_{i2} p \\ \rho u_i w + \delta_{i3} p \\ \varepsilon_{ve,m} u_i \\ (\varepsilon + p) u_i \end{pmatrix} \quad \boldsymbol{\mathcal{F}}_{i,vis} = \begin{pmatrix} 0 \\ -\mathcal{J}_{s,i} \\ \tau_{i1} \\ \tau_{i2} \\ \tau_{i3} \\ -q_{ve,i,m} - e_{ve,m}\mathcal{J}_{m,i} \\ \tau_{i,j}u^j - q_{tr,i} - q_{ve,i} - \sum_{r\neq e} h_r \mathcal{J}_{r,i} \end{pmatrix} \quad \dot{\boldsymbol{\mathcal{W}}} = \begin{pmatrix} 0 \\ \dot{\omega}_s \\ f_{i1} \\ f_{i2} \\ f_{i3} \\ \varepsilon_{ve,m} \\ Q_m \end{pmatrix}$$

$$s \in N_s, m \in N_m$$

Quantity	Model
Shear viscosity	Blottner
Heat conduction	Fourier’s law
Thermal diffusivity	Eucken’s formula
Mixing Rule	Wilke
Diffusion fluxes	Fick’s law

Quantity	Model
V-T energy exchange	Landau-Teller equation
V-T relaxation time	Millikan-White-Park correction
V-V energy transfer	No V-V transfer
Reaction VE energy source	Park TTv model

MHD terms, 2T model

MHD terms: $Re_{mag} = \sigma \mu_0 U L \approx 0.001$ $\quad \boldsymbol{f} = \boldsymbol{j} \times \boldsymbol{B}$ $\quad Q_m = \boldsymbol{jE}$ $\quad \boldsymbol{j} = \sigma \left(\boldsymbol{E} + (\boldsymbol{U} \times \boldsymbol{B}) + \frac{\nabla P_e}{e n_e} \right)$

Two-temperature model

$$E = \frac{1}{2}\rho \sum_i u_i^2 + \sum_{s \neq e} E_{t,s} + \sum_{s \neq e} E_{r,s} + \sum_{s \neq e} E_{v,s} + \sum_{s \neq e} E_{el,s} + E_e + \sum_{s \neq e} \rho h_s^o , \qquad E_{*,s} = \rho_s e_{*,s}$$

Specific energies
Vibrational (1)
Translational (2)
Rotational (3)
Electronic (4)

$$e_{v,s} = R_s \frac{\theta_{v,s}}{\exp\left(\frac{\theta_{v,s}}{T_{ve,s}}\right) - 1}, \quad (1) \qquad e_r = R_s T_{tr}, \quad (3)$$

$$e_t = \frac{3}{2} R_s T_{tr}, \quad (2) \qquad e_e = R_e T_{ve,ref} \quad (4)$$

$$e_{el,s} = R_s \frac{\sum_{i=1}^{\infty} g_{i,s} \theta_{el,i,s} \exp\left(-\frac{\theta_{el,i,s}}{T_{ve,s}}\right)}{\sum_{i=0}^{\infty} g_{i,s} \exp\left(-\frac{\theta_{el,i,s}}{T_{ve,s}}\right)} \quad (5)$$

h_s^o - enthalpy of formation
R_s - specie's gas constant
$\theta_{v,s}$ - characteristic vibrational temperature
$\theta_{el,i,s}$ - characteristic electronic temperature

$$E_{tr} = E_t + E_r$$

$$E_{ve,s} = E_{v,s} + E_{el,s}$$

$$E = \frac{1}{2}\rho \sum_i u_i^2 + E_{tr,s} + E_{ve,s} + E_e + \sum_{s \neq e} \rho h_s^o$$

Energy transfer models

Energy transfer

Fourier Law

$$q_{tr,i} = \sum_s q_{tr,i,s} = \sum_s -\kappa_{tr,s} \frac{\partial T_{tr}}{\partial x_i}; \quad s \in N_s \setminus \{e\} \qquad q_{ve,i} = \sum_s q_{ve,i,s} = \sum_s -\kappa_{ve,s} \frac{\partial T_{ve}}{\partial x_i}; \quad s \in \boldsymbol{N_s} \setminus \{\boldsymbol{e}\}$$

Landau-Teller model for V-T energy modes relaxation

$$Q_{m,V-T} = \rho_m \frac{\partial e_{ve,m}(T_{ve,m})}{\partial t} = \rho_m \frac{e_{ve,m}(T_{tr}) - e_{ve,m}(T_{ve,m})}{\tau_{m,V-T}}, \qquad m \in N_m,$$

Millikan-White V-T relaxation time model

$$\tau_{m,V-T} = \frac{\sum_{s \neq e} X_s}{\sum_{s \neq e} \frac{X_s}{\tau_{m-s,V-T}}}, \qquad m \in N_m$$

$$\tau_{m-s,V-T} = \frac{1}{p} \exp\left[A_{m,s}\left(T_{tr}^{-\frac{1}{3}} - B_{m,s}\right) - 18.42\right] + \frac{1}{c_m \sigma_{v,m} n_{m,s}}$$

c_m - average molecular velocity
$\sigma_{v,m}$- collision cross-sections
$n_{m,s}$ - numerical density of colliding particles pairs

$$A_{m,s} = 1.16 \times 10^{-3} \sqrt{M_{m,s}} \theta_{v,m}^{\frac{4}{3}}, \qquad c_m = \sqrt{\frac{8 R_m T_{tr}}{\pi}}$$

$$B_{m,s} = 0.015 M_{m,s}^{\frac{1}{4}} \qquad M_{m,s} = \frac{M_m M_s}{M_m + M_s}$$

Chemistry model

Atmosphere	
Earth	C. Park; Review of Chemical-Kinetic Problems of Future NASA Missions, I: Earth Entries; JOURNAL OF THERMOPHYSICS AND HEAT TRANSFER; Vol.7, No.3, July.-Sept. 1993
Mars	C. Park, J. T. Howe, R. L. Jaffe and G. V. Candler Review of Chemical-Kinetic Problems of Future NASA Missions, II: Mars Entries; JOURNAL OF THERMOPHYSICS AND HEAT TRANSFER; Vol.8, No.1, Jan.-March 1994

	Earth	Mars 1	Mars 2	N_2
Number of reactions	19	18	33	4
Dissociation	3	6	8	1
Exchange	2	4	14	0
Associative ionization	3	2	3	1
Charge exchange	9	4	6	0
Electron impact ionization	2	2	2	1
Electron impact dissociation	1	0	0	1

Arrhenius equation

$$k = Ae^{-\frac{E_a}{RT}}$$

$$T = \sqrt{T_t T_v}$$

Components:

Earth atmosphere:

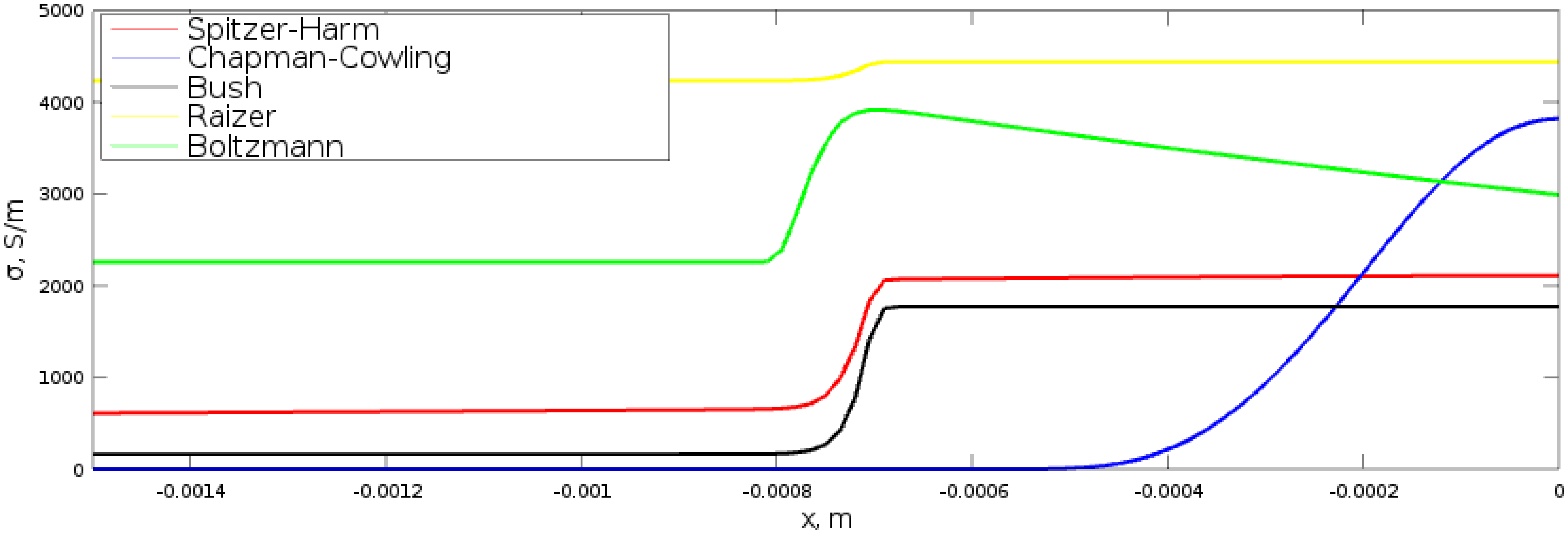
N_2, O_2, N, O, NO, N_2+, O_2+, N+, O+, N+, e-

Mars atmosphere

N_2, **O_2**, NO, **CO_2**, CN, **C_2**, NCO, N_2+, **O_2+**, NO+, CN+, **CO+**, **C_2+**, N, **O**, **C**, Ar, N+, **O+**, **C+**, Ar+, **e-**

Electric conductivity models

	Boltzman	Spitzer-Harm	Chapman-Cowling	Bush	Raizer
Formula	$\frac{e^2 n_e \tau_e}{m_e}$	$\frac{1.56 \times 10^{-4} \times T^{\frac{3}{2}}}{\ln\left(1.23 \times 10^4 \times \frac{T^{\frac{3}{2}}}{\sqrt{n_e}}\right)}$	$3.34 \times 10^{-12} \frac{\alpha}{Q\sqrt{T}}$	$\sigma_0 \left(\frac{T}{T_0}\right)^n$	$83 \times e^{-\frac{36000}{T}}$
Characteristic value $[\Omega^{-1} * \mathrm{cm}^{-1}]$	30	4.4	4317	210	5131
Computation time (10 000 cells grid)	3.8e-01	4.7e-03	8e-03	4e-03	4.3e-03



Advanced conductivity model, Numerical Schemes

$$\sigma = \frac{e^2 n_e \tau_e}{m_e}, \qquad \tau_e^{-1} = \sum_k \tau_{ek}^{-1}, \quad \tau_{en}^{-1} = n_n v_e \Delta_{en},$$

$$\tau_{ei}^{-1} = n_e c_e \Delta_{ei}, \qquad \Delta_{ei} = \frac{4\pi}{9}\frac{e^4 \Lambda}{(kT_e)^2},$$

$$\Lambda = \ln\left[\frac{3}{2\sqrt{\pi}}\frac{(kT_e)^{\frac{3}{2}}}{e^3 n_e^{\frac{1}{2}}}\right], \qquad c_e = \sqrt{\frac{3kT_e}{m_e}},$$

Numerical Schemes	
Term	**Scheme**
Time stepping	Euler
Fluxes	Kurganov
Gradient	Gauss linear (cellLimited Gauss linear 1)
Divergence • Species diffusion • Velocity	Gauss limitedLinear 1 • Gauss linear • Gauss linear
Laplacian	Gauss linear corrected
interpolation	vanLeer
snGradSchemes	corrected

«Electron-heavy particle» collision rate evaluation

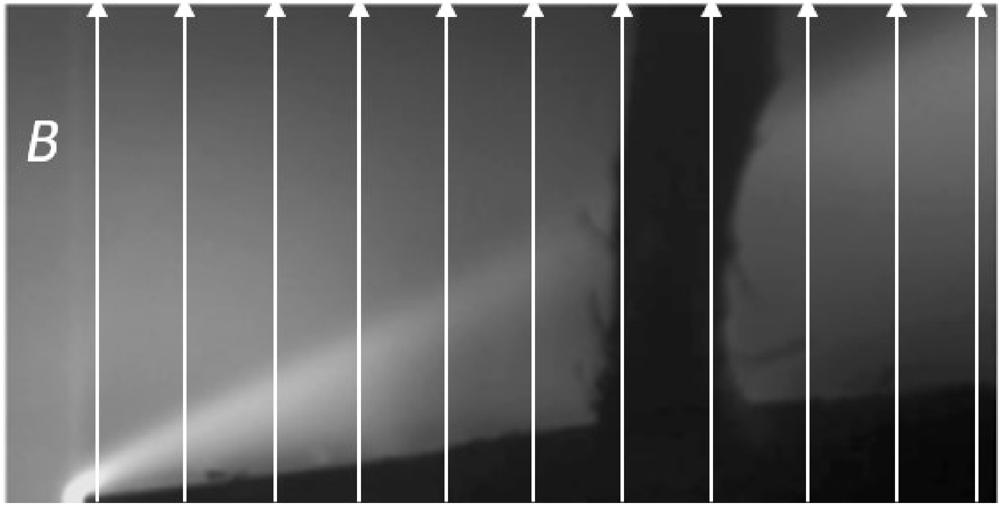
$$\Delta = 4.398 \times 10^{-10} \ln\left(\frac{\Lambda}{T^2}\right)$$

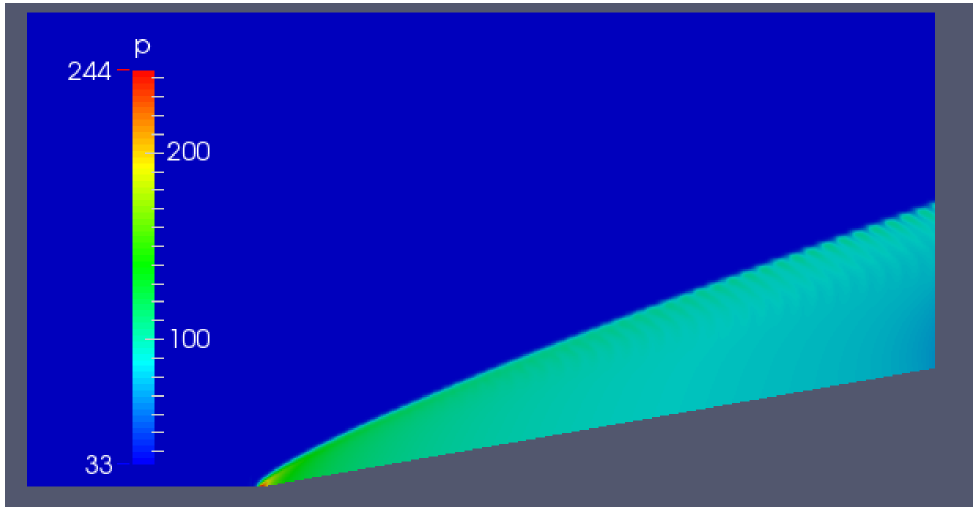
$$\Delta_{ei} = \frac{4\pi}{9}\frac{e^4\Lambda}{kT_e^2};$$

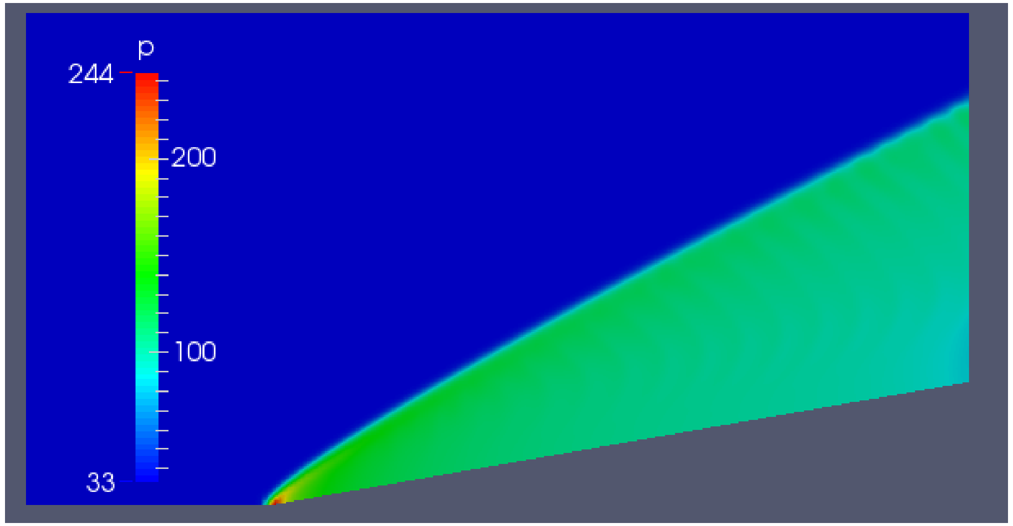
$$\Delta_{en} \sim 4 \cdot 10^{-20} m^2$$

Gupta, R.; Yos, J.M.; Thompson, R.A.; Lee, K.P. A Review of Reaction Rates and Thermodynamic and Transport Properties for an 11-Species Air Model for Chemical and Thermal Nonequilbrium Calculations

MHD terms test (wedge geometry)





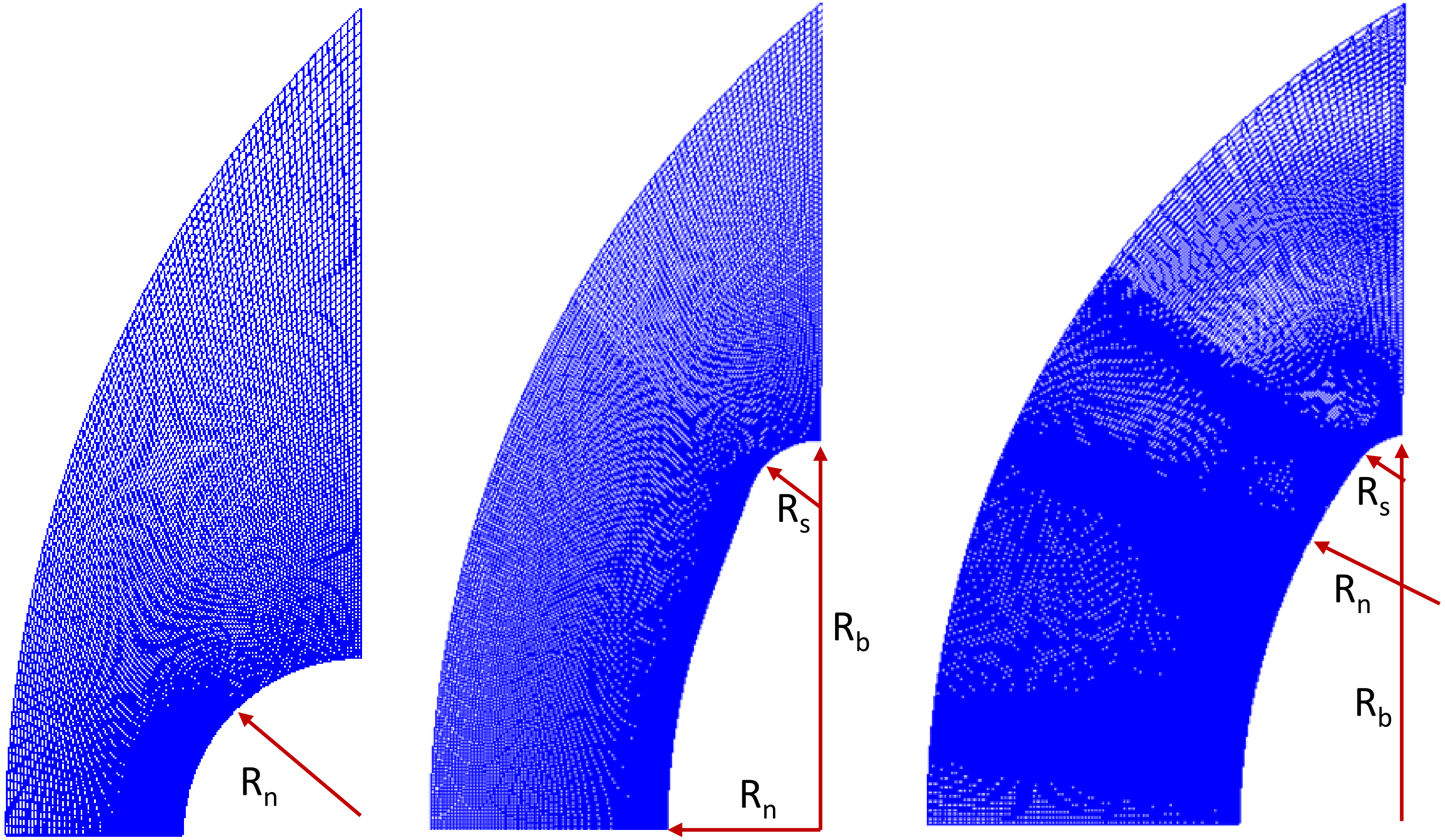


Gas	$U_\infty \left(\frac{m}{s}\right)$	$p_\infty(Pa)$	$T_\infty(K)$	$B(T)$
Air	5000	33	575	2

В. А. Битюрин, А. Н. Бочаров, Экспериментальные и численные исследования МГД-взаимодействия в гиперзвуковых потоках, ТВТ, 2010, том 48,дополнительный выпуск, 44–55

Case geometries

1. Cylinder	2. MSL	3. Spherical Section

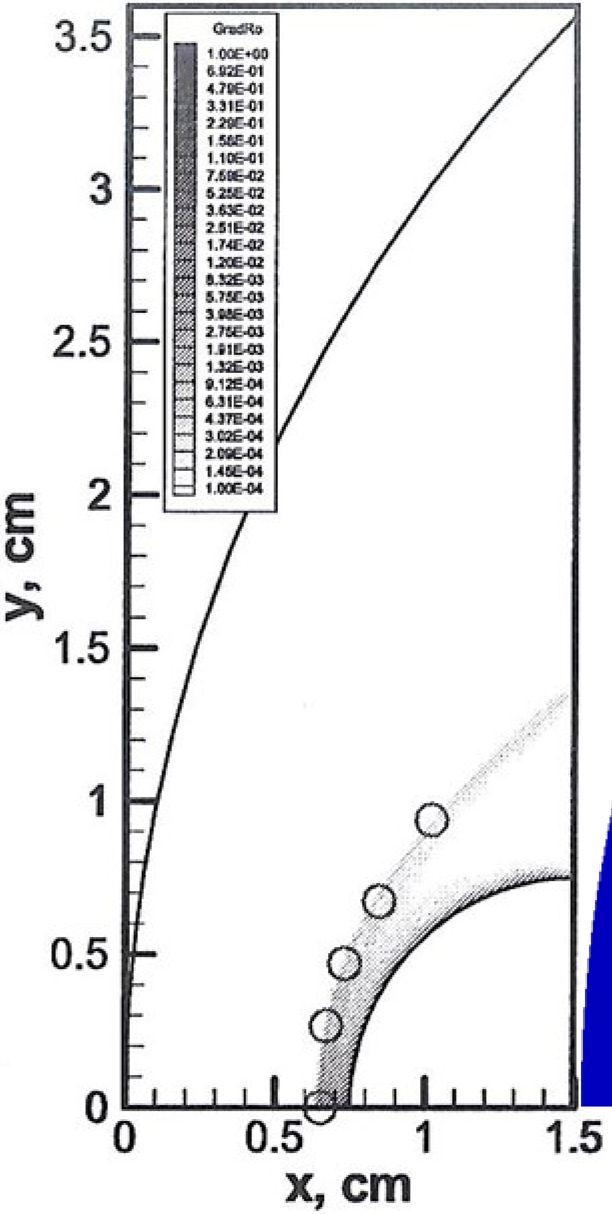


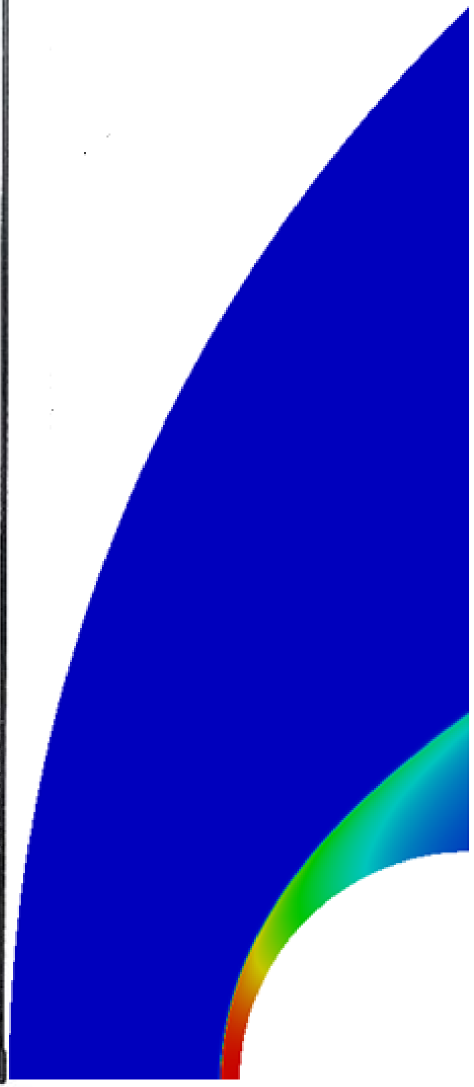
	Cylinder	MSL	Spherical section
R_s	-	0.125	0.3
R_b	-	2.25	2.5
R_n	0.0075	1.125	3

Cylinder test case

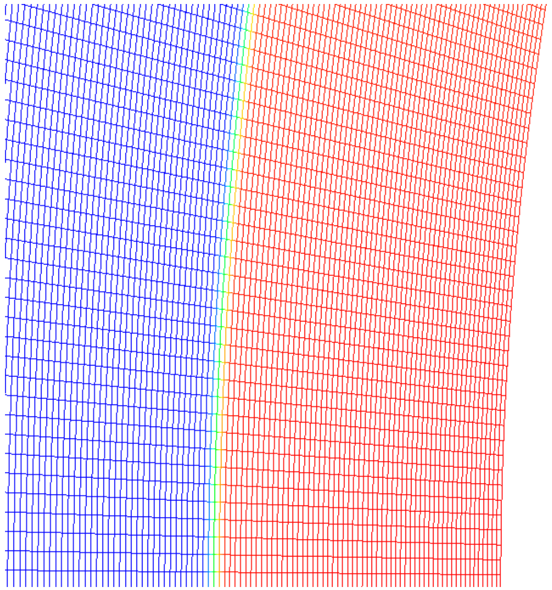
Flow parameter	
Parameter	**Value**
Gas	N_2
U_∞	10300 m/s
p_∞	13 000 Pa
T_∞	3030 K
ρ_∞	0.0014 kg/m^3

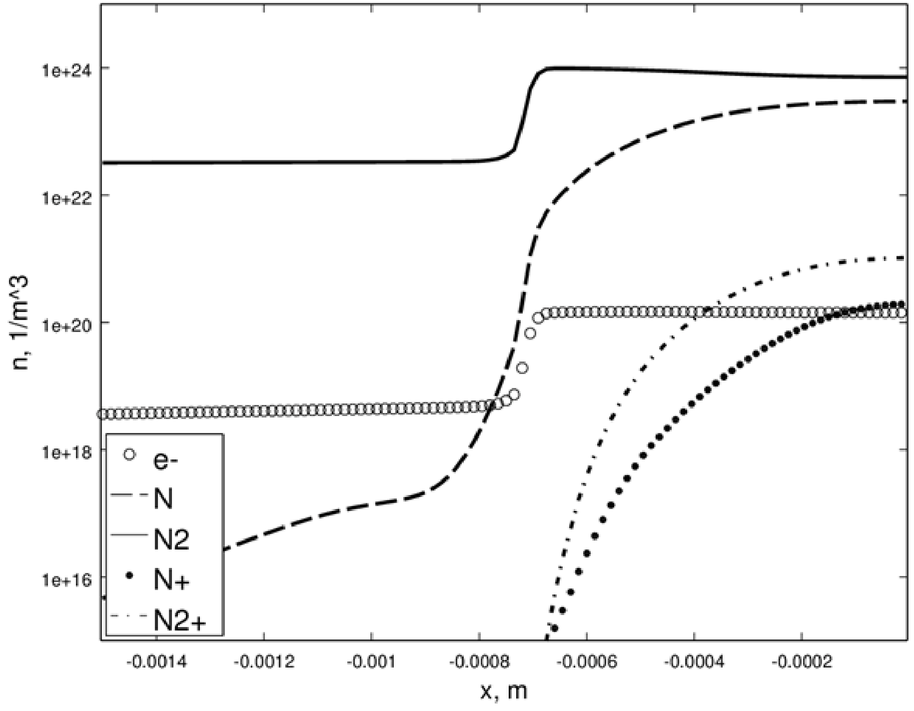
Суржиков С. Т. Расчетное исследование аэротермодинамики гиперзвукового обтекания затупленных тел на примере анализа экспериментальных данных //М.: ИПМ им. АЮ Ишлинского РАН. – 201



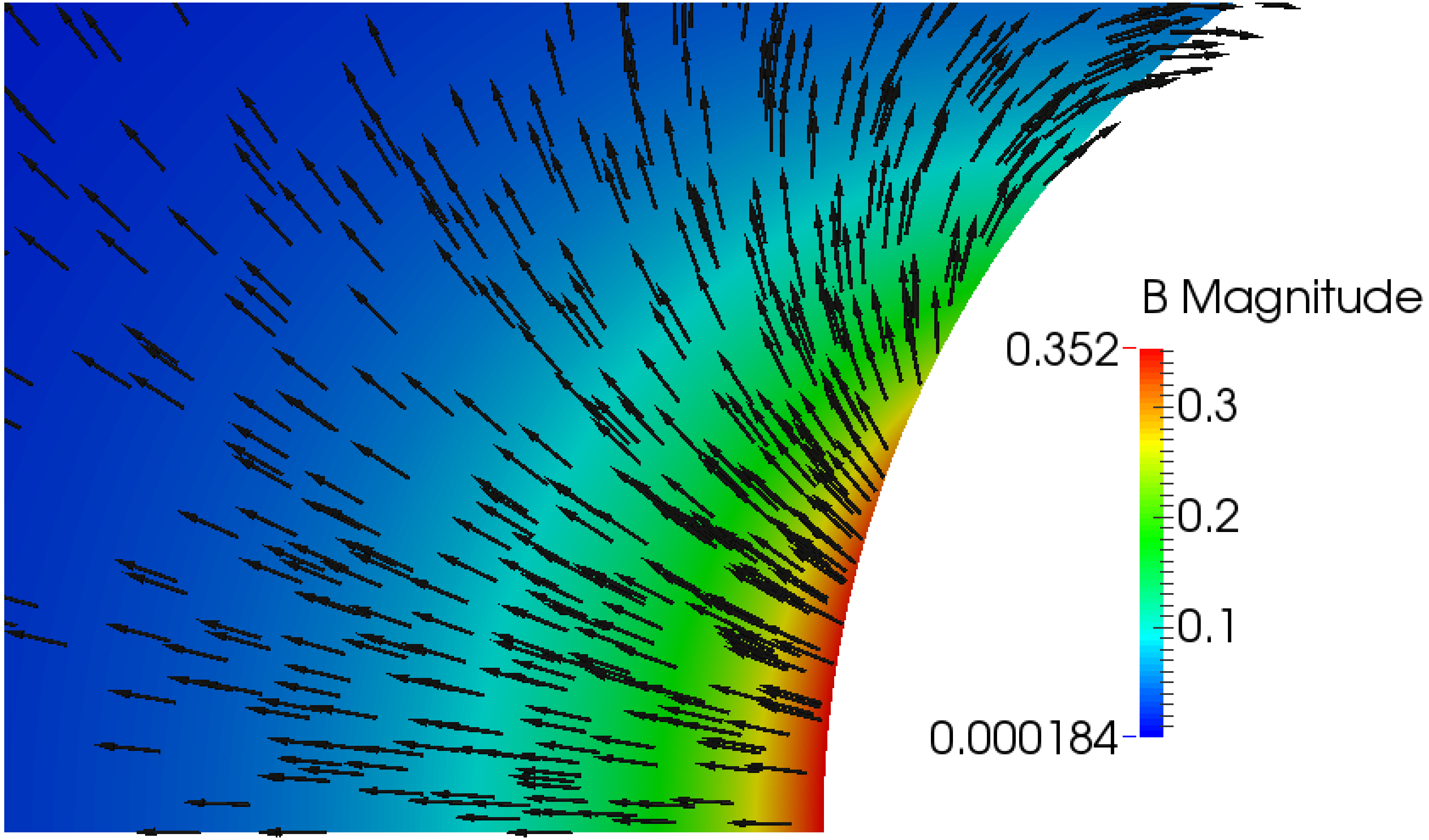


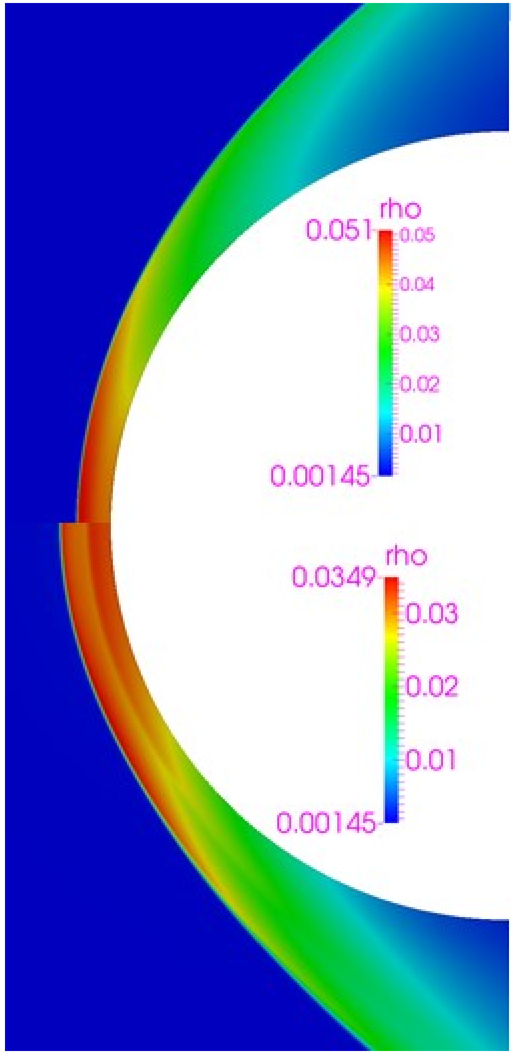
Mesh fragment (~60 cells within the shock layer)



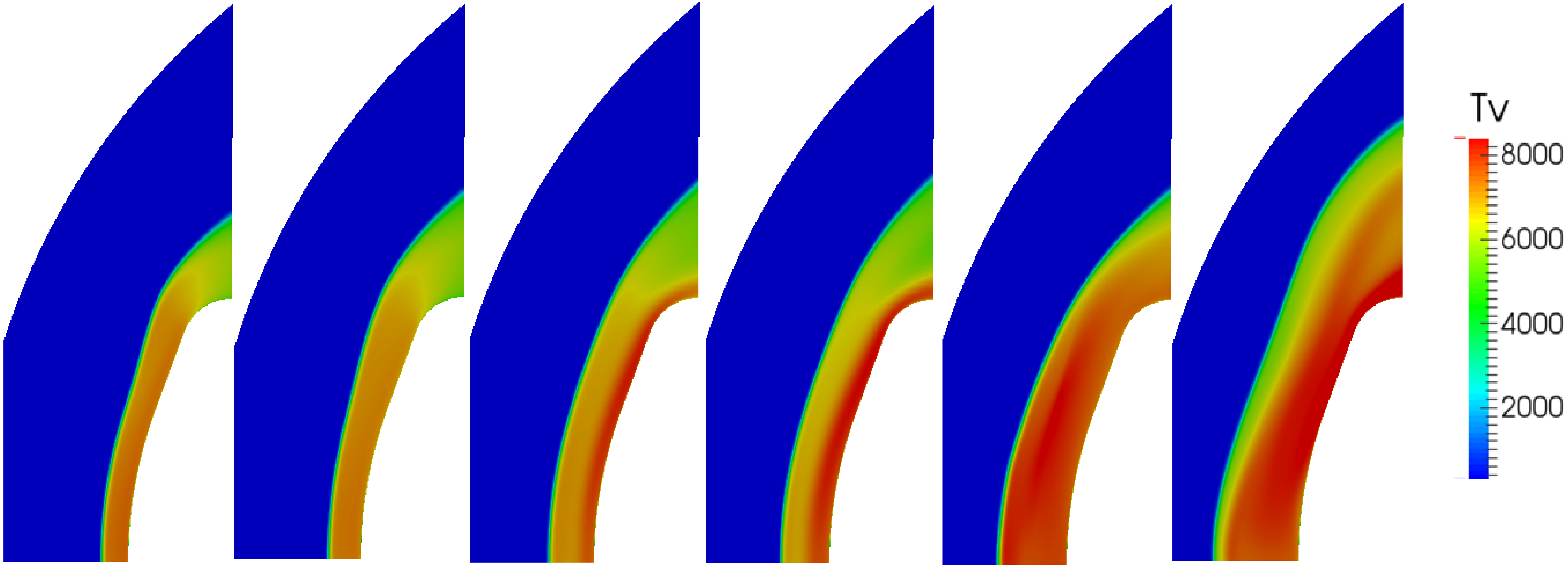


Cylinder test case with magnetic field





Conductivity model effect

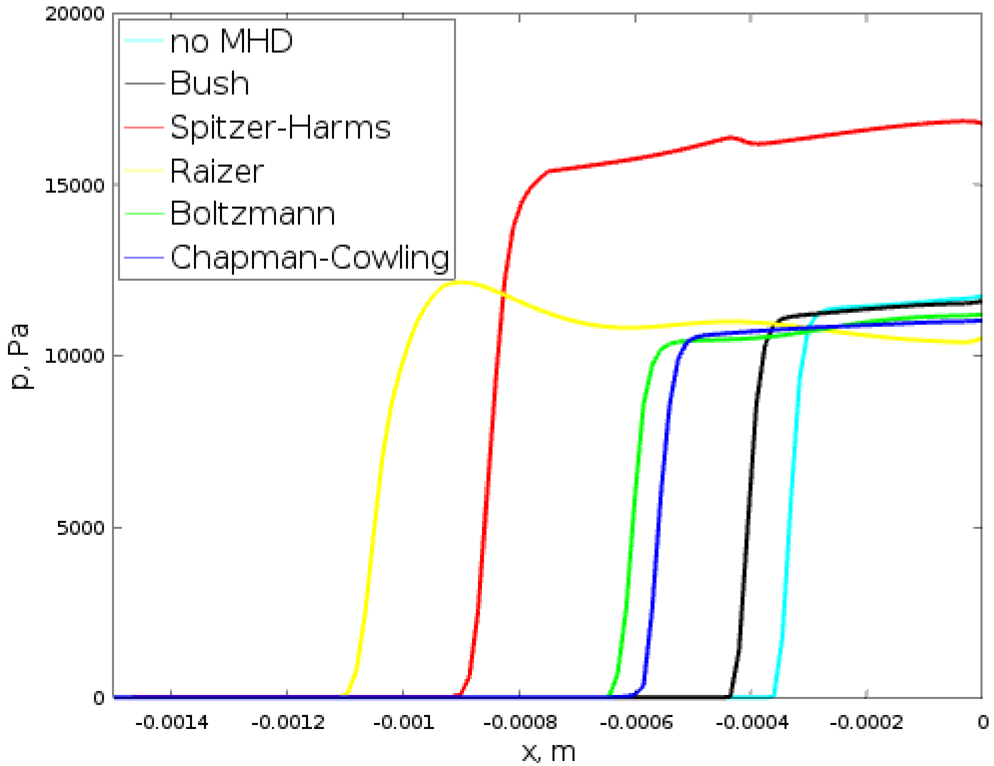


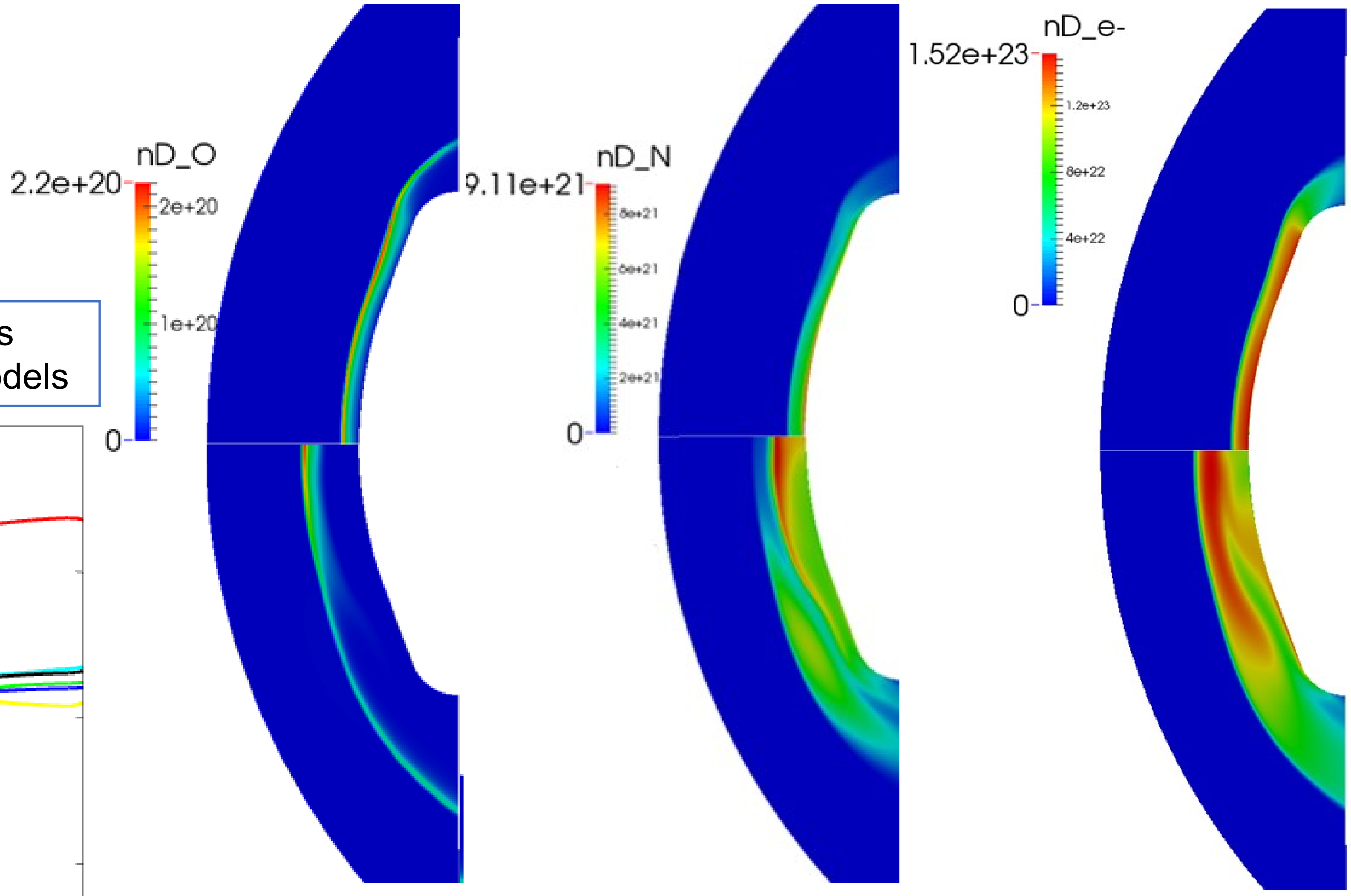
no MHD	Bush	Chapman-Cowling	Boltzmann	Spitzer-Harm	Raizer

Flow around entry vehicle model

Parameter	Value
U_∞	5411.2 m/s
T_∞	182 K
ρ_∞	0.001013 kg/m³
M	24.2

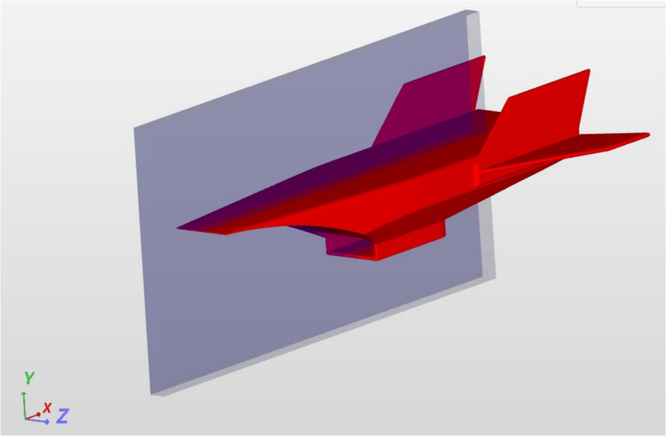
Pressure profiles on stagnation lines calculated with different conductivity models





Scramjet inlet flow control

NASA X-43

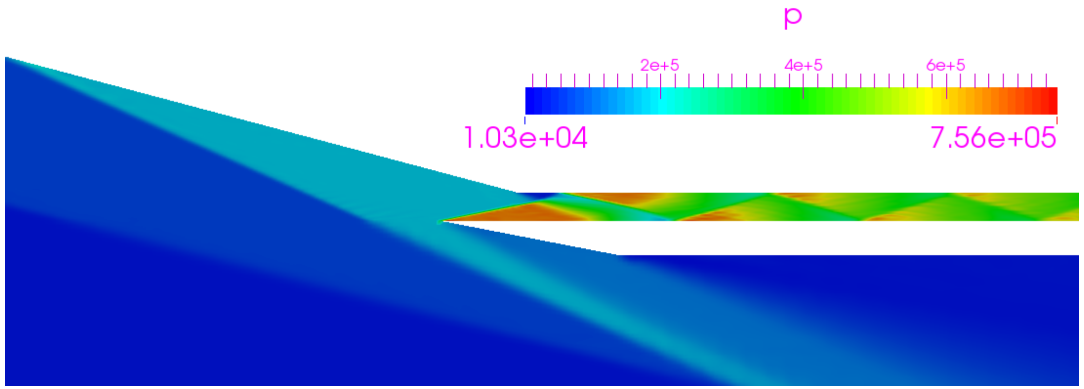


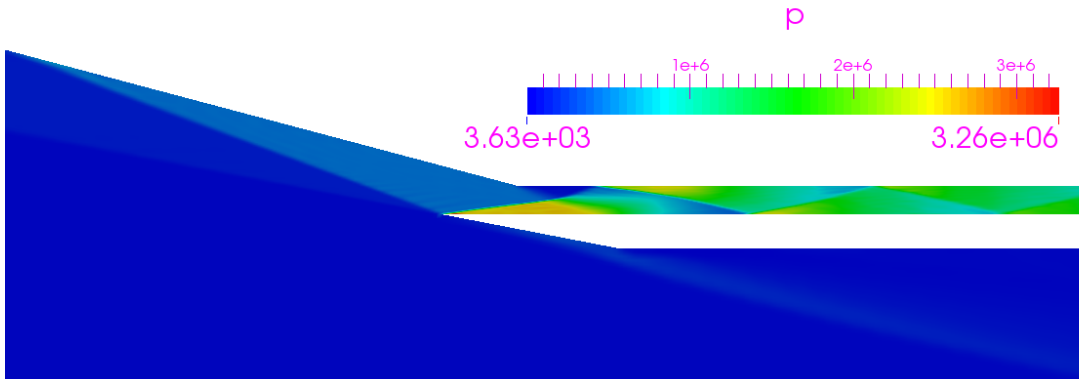
Central section of
a scramjet mesh

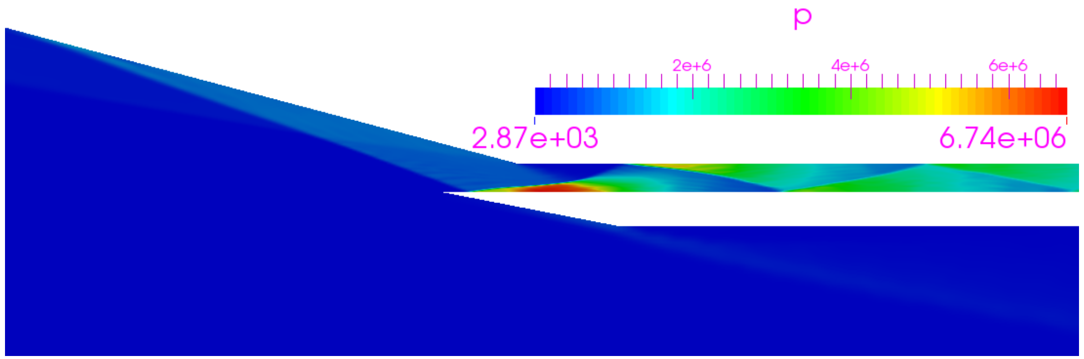
Parameter	Value
p	26436 Pa
T	223 K
ρ	0.412707 kg/m^3
C	300 m/s
M	6, 10 (design), 14

Right: Flow into scramjet engine M =

- 0.6 M_{design},
- M_{design},
- 1.4 M_{design}



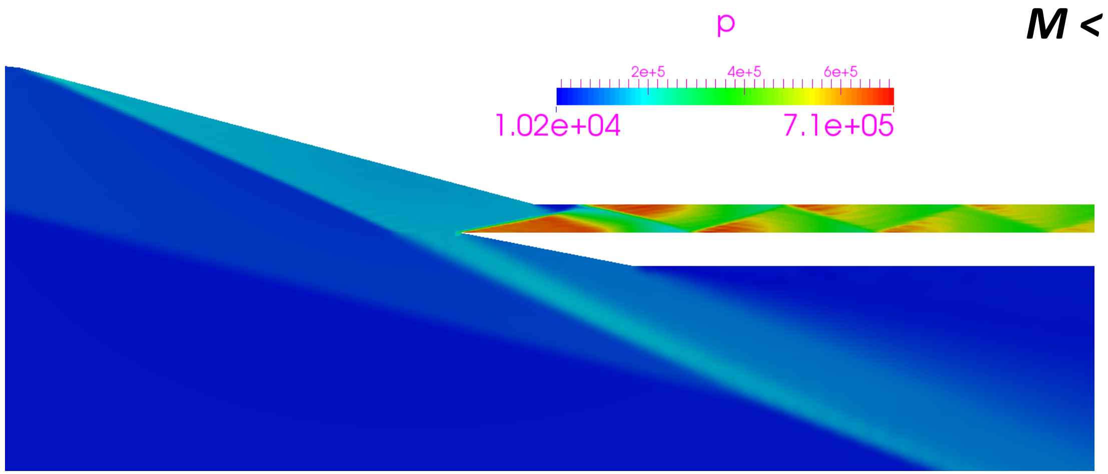


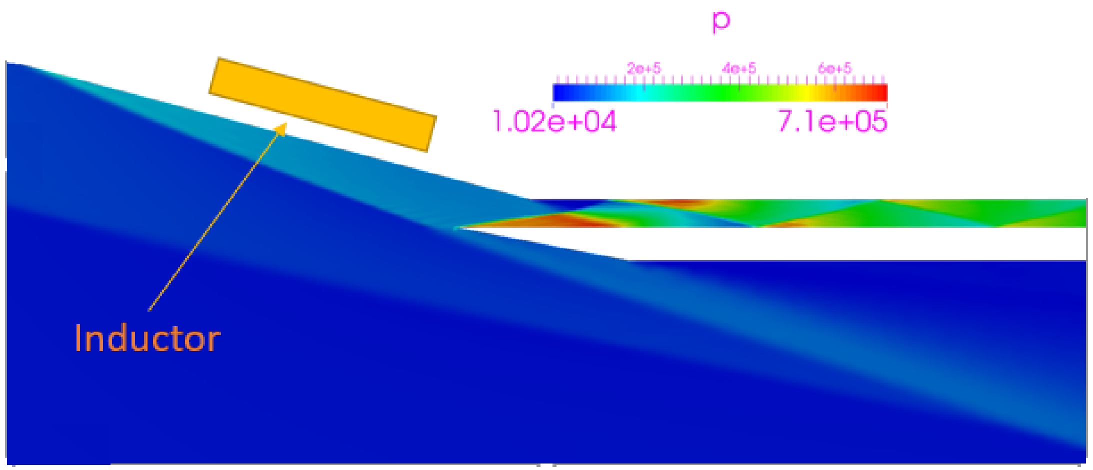


Scramjet inlet flow control

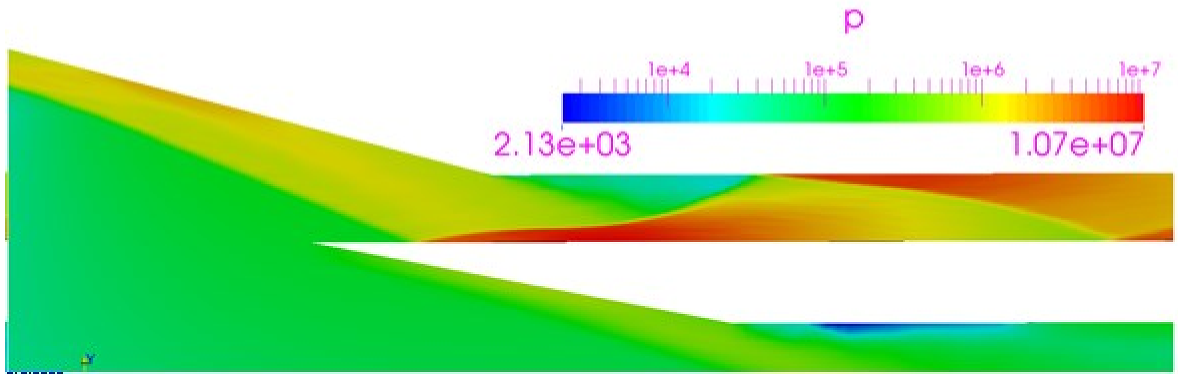
Control of the pressure jump at the scramjet engine inlet

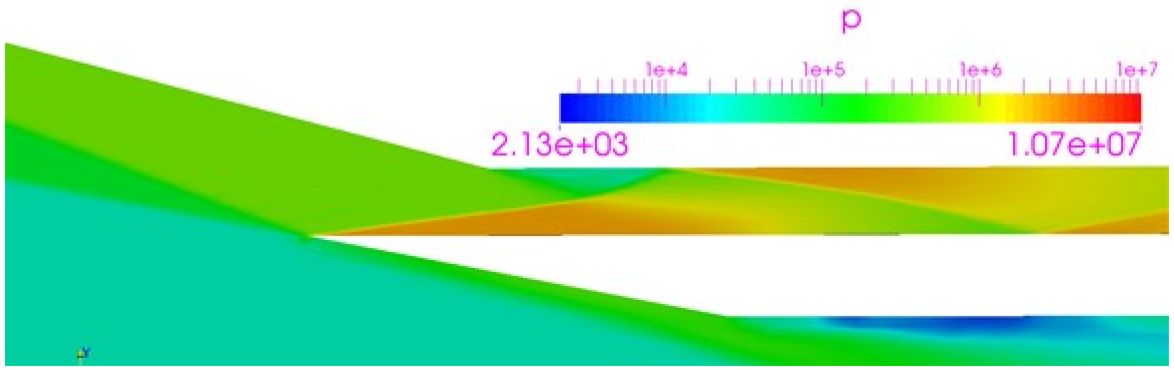
***M* < *M*$_{design}$**





***M* > *M*$_{design}$**





Future R&D goals and prospects

Development goals						
hy2MhdFoam						Hybrid CFD-DSMC solver
MHD					**Radiation**	Future agenda
Basics	Conductivity models	Hall effect	Ion slip	Artificial ionization	Basic and P1 model implemented and is being tested	
Completed	Completed	Testing	Under development	Early development stage		
Research goals						

- Demonstrating the potential of MHD flow control for hypersonic regimes for 2D symmetric flows
- Evaluating the required energy input for the desired effect
- Optimizing the configuration of MHD flow control system to maximize the efficiency and flexibility

Source code (available in January 2018): ***https://github.com/vincentcasseau/hyStrath***