Software for Wavelet-based Adaptive Grids

Artem Semakin
Bauman Moscow State Technical University

2017 Ivannikov ISPRAS Open Conference
December 1, 2017
Moscow, Russia
Motivation

- Global atmospheric transport is a very complex multiscale phenomenon. Significant spatial scales of the global atmospheric flow range from small scale convection (~100m or less) to large scale circulation (~1000km). Temporal chemical scales range from fractions of a second to a year or longer.

- However, typical spatial resolution currently used for numerical modeling of atmospheric transport is 0.5°-4°. This relatively crude resolution results in significant numerical diffusion. As a result, numerical error may exceed 90%.

- Therefore, it is necessary to develop a new multiscale numerical technique that allows us to resolve all important spatial scales with presently available computational resources and, hence, produces much more accurate solution than conventional non-adaptive numerical algorithms.

- Wavelet-based Adaptive Mesh Refinement (WAMR)!
Fukushima I Nuclear Disaster
March 11, 2011

Released radioactive materials:
1) Iodine-131,
2) Caesium-134,
3) Caesium-137.

The radioactive isotopes reached
1) the east coast of Russia in 3 days
2) the west coast of the USA in 5 days.

Deposition of Cesium on May 26, 2011
(Japan Science Ministry)
Currently Developed Algorithms

Adaptive Mesh Refinement

Block-based grid
- PARAMESH
- AMRCLAW
- AMROC
- AMR++

Element-based grid
- AMRCLAW
- AMROC
- AMR++
- Gerris
- WAMR

Cell-based grid
- PARAMESH
- AMRCLAW
- AMROC
- AMR++
- Gerris

Vertex-based grid
- Gerris
- WAMR
Advection-Diffusion-Reaction Equation

\[ \frac{\partial c_i}{\partial t} + u \cdot \nabla c_i = \nabla \cdot D \nabla c_i + \omega_i, \quad i = 1, \ldots, N \]

Net chemical production rate is defined as

\[ \omega_i = \sum_{l=1}^{N} k_l^i c_l + \sum_{l, j=1}^{N} k_{lj}^i c_l c_j , \]

The system of equations is extremely stiff, nonlinear and involves a large number of species. Typical simulations involve \( \sim 100 \) species and \( \sim 1000 \) chemical reactions on different scales: from local to intercontinental – in space, from seconds to days – in time.
Adaptive Wavelet Basis and Grid

\[ c = \sum_{k} b_{k} \varphi_{k} + \sum_{l=0}^{\infty} \sum_{k} d_{l,k} \psi_{l,k} \]

\[ P^{\varepsilon} c = \sum_{k} b_{k} \varphi_{k} + \sum_{l=0}^{J} \sum_{\left| d \right| > \varepsilon} d_{l,k} \psi_{l,k} \]

Sparse wavelet representation
Adaptive Grid Structure
Advection Equation for an Inert Plume

\[ \frac{\partial c}{\partial t} + u \cdot \nabla c = 0 \]

\[ \frac{dc}{dt} = 0 \]

\[ c = \text{const} \quad \text{along the trajectory} \quad \frac{dr}{dt} = u(r, t) \]

\[ t_0 \quad c_1 = \text{const} \]
\[ c_2 = \text{const} \]
\[ c_3 = \text{const} \]

\[ t_1 \]
Evolution of a plume at different time moments

2D Atmospheric Flow
Stationary Velocity Field

Evolution of a plume at different time moments
2D Non-Uniform Multilevel Grid

Evolution of 2D Transpacific plume
Grid Dynamics

Number of active points for the 2D adaptive grid

Increase of grid size $N$ with resolution $I$

Compression ratio $\sim 1000$ for 2D flows!!!!
Thank you