Tool for Behavioral Analysis of Well-Structured Transition Systems

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Abstract. Well-structured transition systems (WSTS) became a well-known tool in the study of concurrency systems for proving decidability of properties based on coverability and boundedness. Each year brings new formalisms proven to be WSTS systems. Despite the large body of theoretical work on the WSTS theory, there has been a notable gap of empirical research of well-structured transition systems. In this paper, the tool for behavioural analysis of such systems is presented. We suggest the extension of SETL language to describe WSTS systems (WSTSL). It makes the description of new formalisms very close to the formal definition. Therefore, it is easy to introduce and modify new formalisms as well as conduct analysis of the behavioural properties without much programming efforts. It is highly convenient when a new formalism is still under active development. Two most studied algorithms for analysis of well-structured transition systems behavior (backward reachability and the Finite Reachability Tree analyses) have been implemented; and, their performance was measured through the runs on such models as Petri Nets and Lossy Channel Systems. The developed tool can be useful for incorporating and testing analysis methods to formalisms that occur to be well-structuredness transition systems.

Keywords: formal verification; infinite systems; well structured transition systems; Petri nets

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1. Introduction

Formal verification provides researchers and developers with approaches that are widely-used for proving that a program satisfies a formal specification of its behavior. These methods are highly demanded in the software and hardware engineering, as they provide appropriate level of systems reliability; which, in most cases, cannot be ensured by simulation.

One of the most common technique of formal verification is model checking or property checking. It involves algorithmic methods that are applied to check satisfiability of a logic formula used for the representation of the specification and the model of a system. The main advantage of model checking is considered to be the fact that it enables almost completely automatic process of verification. Model checking proved to be effective in practice for analysis of finite-state systems [1]; however, in case of systems with infinite state space the situation is more complicated because exhaustive search, which is usually used by verification tools, cannot be applied directly.

In order to deal with infinite-state systems Finkel proposed the idea of well-structured transition systems (WSTS) in 1987 [2]. “These are transition systems where the existence of a well-quasi-ordering over the infinite set of states ensures the termination of several algorithmic methods. [3]” The suggested model has provided researchers with an abstract generalization of several models (e.g. Petri nets, lossy channel systems and timed automata). Therefore, the results obtained from the analysis of such a generalized model can be also applied to these specific models.

The WSTS analysis can be used to solve, for instance, covering, termination, inevitability and boundedness problems. However, the application of the WSTS analysis is hampered by the necessity of implementing algorithms and data structures to support the analysis for each new formalism. In this work, the tool that can be used for analysis of WSTS is presented. We introduce the WSTSL language - modification of SETL language [13,14] – set-theoretical programming language. The language provides the user with opportunity to define the structure of analyzed system as close to the original formal definition as possible. After definition of the formalism, it is immediately possible to run backward reachability method [4] or the Finite Reachability Tree [5] on it. It is convenient for computer science researcher to postpone the implementation phase after what-if experiments.

The rest of the paper is organized as follows. The second section describes WSTS’s basic terms and underlying concepts. The third section provides the description of two used algorithms (the backward reachability method and the Finite Reachability Tree). The forth section presents the architecture of the developed analysis tool. The fifth section shows how the developed tool is used for the analysis of Petri nets and provides performance analysis results. The sixth section summarizes and provides possible applications of the study for the future research.

2. Well-Structured Transition Systems

The definition of well-structured transition systems (WSTS) was proposed by Finkel in [2]. It is based on the two main concepts: transition systems (TS) and well-quasi-orderings between the states of these systems.
Transition system (TS) is one of the most widely-used models for formal description of the behavior of different systems. A transition system is defined by a structure $TS = (S, \rightarrow, \subseteq)$ where $S$ is a set of states, and $\rightarrow \subseteq S \times S$ is any set of transactions [3]. TS can be also supplemented by other structures such as initial states, labels for transitions, durations or causal independence relations [3]; however, for the consideration of the concept of WSTS using of set of states along with set of transactions is sufficient.

A binary relation $\subseteq$ on a set $X$ is called preorder or quasi-ordering (qo) if it is reflexive and transitive. So for any $a, b, c \subseteq X$ we have:

- $1)$ $a \subseteq a$ (reflexivity);
- $2)$ if $a \subseteq b$ and $b \subseteq c$ then $a \subseteq c$ (transitivity).

**Definition 1.** A well-quasi-ordering (wqo) is a qo in which for every infinite sequence of elements $x_0, x_1, x_2, x_3, \ldots \subseteq X$ there exist such indices $i < j$ that $x_i \subseteq x_j$ [3, 6]. According to [7], there are a range of equal definitions of wqo; however, the definition given above is generally used in papers on WSTS.

**Definition 2.** A well-structured transition system (WSTS) is a transition system $TS = (S, \rightarrow, \subseteq)$ equipped with a preorder $\subseteq \subseteq S \times S$ between states such that the two following conditions hold:

- $1)$ well-quasi-ordering: $\subseteq$ is a wqo, and
- $2)$ compatibility: $\subseteq$ is (upward) compatible with $\rightarrow$, i.e. for all $s_1 \subseteq t_1$ and transition $s_1 \rightarrow s_2$, there exists such a sequence of transitions $t_1 \rightarrow t_2$ that $s_2 \subseteq t_2$ [3].

$Succ(s)$ denotes the set $\{s' \in S \mid s \rightarrow s'\}$ of immediate successors of $s$. Likewise, $Pred(s)$ denotes the set $\{s' \in S \mid s' \rightarrow s\}$ of immediate predecessors.

An upward-closed set is any set $I \subseteq X$ such that $y \geq x$ and $x \in I$ entail $y \in I$. A basis of an upward-closed $I$ is a set $I^b$ such that $I = \bigcup_{x \in I^b} \uparrow x$, where $\uparrow x = \{y \mid y \geq x\}$.

### 3. Algorithms

#### 3.1 Backward Reachability Method

Backward reachability method proposed by Abulla et al. in [4] is intended to solve the covering problem which is to decide, given two states $s$ and $t$, whether starting from $s$ it is possible to reach a state $t' \geq t$. This is essentially one of set-saturation methods termination of which relies on the lemma that says that any increasing sequence of upward-closed sets ($I_0 \subseteq I_1 \subseteq I_2 \subseteq \cdots$) eventually stabilizes (i.e. there is such a $k \in N$ that $I_k = I_{k+1} = I_{k+2} = \cdots$) [3].

Assume there is some WSTS $TS = (S, \rightarrow, \subseteq)$ and some upward-closed set of states $I \subseteq S$. Backward reachability method on the each $j$-th step generates the set of states from which $I$ can be reached by a sequence at most $j$ transitions [4].

More strict generalization was suggested by Finkel and Schnoebelen in [3], where it involves computing $Pred^*(I)$ as the limit of the sequence $I_0 \subseteq I_1 \subseteq \cdots$ where $I_0 = \text{def} I$ and $I_{n+1} = \text{def} I_n \cup Pred(I_n)$.

**Definition 3.** A WSTS has effective pred-basis if there exists an algorithm accepting any state $s \in S$ and returning $pb(s)$, a finite basis of $\uparrow Pred(s)$.

The covering problem is decidable for WSTS if it has effective pred-basis and decidable $\subseteq$. The proof of this statement is given in [3]. Essentially, it is said that if there is a sequence $K_0, K_1, \ldots$ with $K_0 = \text{def} I^b$ (finite basis of $I$), $K_{n+1} = \text{def} K_n \cup pb(K_n)$ and $m$ is the first index such that $\uparrow K_m = \uparrow K_{m+1}$, then $\uparrow I = \uparrow K_0$. By decidability of $\subseteq$, it is possible to check whether $s \in \uparrow Pred^*(\uparrow t)$.

### 3.2 Finite Reachability Tree

The Finite Reachability Tree belongs to tree-saturation methods which represent methods that consider all possible computations inside a finite tree-like structure [3]. It is also called the forward analysis method, in contrast to the backward analysis. Essentially, it is based on the ideas proposed by Karp and Miller in [5].

Assume there is some WSTS $TS = (S, \rightarrow, \subseteq)$. For any state $s \in S$, the Finite Reachability Tree is such a finite directed graph (tree) that:

- $1)$ nodes of the tree are labeled by states of $S$;
- $2)$ nodes are either dead or live;
- $3)$ the root node is a live node $n_0$, labeled by $s$ (written $n_0 : s$);
- $4)$ dead nodes have no child nodes;
- $5)$ a live node $n : t$ has one child $n' : t'$ for each successor $t' \in Succ(t)$;
- $6)$ if along the path from the root node $n_0 : s$ to some node $n' : t'$ there exists a node $n : t (n \neq n')$ such that $t \leq t'$, we say that $n$ subsumes $n'$, and then $n'$ is a dead node [3, 6].

The Finite Reachability Tree is effectively computable if $S$ has (1) a decidable $\subseteq$, and (2) $Succ$ mapping is computable [3]. All paths in the finite reachability tree are finite as any infinite path would include a covering node [6].

This algorithm can be applied to termination, inevitability, and boundedness problems (see [3] for details).

#### 4. Proposed Architecture

The general structure of the architecture of the developed tool is illustrated in Fig. 1. It consists of two main parts: Well-Structured Transition Systems Language (WSTSL) and WSTS Analyzer. Also there are four input parameters that are set by the user through WSTSL.
Afterwards, the WSTS Analyzer runs the selected algorithm on the specified system and generates report which format depends on the choice of the algorithm.

5. Experiment

5.1 Petri Net

The applicability of the proposed approach could be demonstrated by an example with common well-structured transition system called Petri net. The classical definition of this model is the following.

Definition 4. A Petri net \((P,T,F,W)\) is a 4-tuple where

- \(P\) and \(T\) are disjoint finite sets of places and transitions, respectively;
- \(F \subseteq (P \times T) \cup (T \times P)\) is a set of arcs;
- \(W : F \rightarrow \mathbb{N} \setminus \{0\}\) – an arc multiplicity function, that is, a function which assigns every arc a positive integer called an arc multiplicity or weight.
- A marking of a Petri net \((P,T,F,W)\) is a multiset over \(P\), i.e. a mapping \(M : P \rightarrow \mathbb{N}\). By \(M(N)\) we denote the set of all markings of the P/T-net \(N\).
- We say that a transition \(t\) in the P/T-net \(N = (P,T,F,W)\) is active in marking \(M\) if for every \(p \in \{(p,t) \in F\}\): \(M(p) \geq W(p,t)\). An active transition may fire, resulting in a marking \(M'\), such as for all \(p \in P\):\(M'(p) = M(p) - W(p,t)\) if \(p \in \{(p,t) \in F\}\), \(M'(p) = M(p) - W(p,t) + W(t,p)\) if \(p \in \{(p,t) \in F\}\) and \(M'(p) = M(p)\) otherwise.

For simplicity’s sake, we consider here the Petri net which arcs can only have multiplicity 1.

For the experiment the Petri net illustrated in Fig. 2 will be considered.

Fig. 2. Instance of the Petri net for consideration in the experiment

First of all, the general structure of the Petri net model described above should be defined by means of WSTL (Fig. 3).
type P : set of int;
type T : set of int;
type PT(P1:P, T1:T) : set of [from P1, from T1];
type TP(P1:T, P1:P) : set of [from T1, from P1];
type M(P1:P) : map <from P1, int>;
type PN(P1:P, T1:T, 
    PT1:PT, TP1:TP) : [P1,T1,PT1,TP1];

Fig. 3. General structure of Petri net in WSTSL

Secondly, we describe the specific Petri net instance in WSTSL (Fig. 4). PT1 and TP1 represent the arcs from places to transitions and vice versa, respectively. In tuples, defining arcs, the corresponding transition goes first for the convenience in description of Succ and Pred function as it will be seen below.

var P1:P = {"P1","P2","P3","P4"};
var T1:T = {"T1","T2"};
var PT1:PT(P1,T1) = {["T1","P1"],["T2","P2"],
    ["T2","P3"];
var TP1:TP(T1,P1) = {["T1","P2"],["T1","P3"],
    ["T2","P1"],["T2","P4"];

Fig. 4. Description of the specific Petri net instance in WSTSL

Then, a well-quasi-ordering should be described (Fig. 5). As it is shown in [3], the inclusion ordering (M ⊆ M’ when M(p) ≤ M’(p) for every place) is a wqo and it is known as Dickson’s lemma [9]. Operator forall iterator | test generates a boolean value true if the condition test is met for each step in iterator and a boolean value false otherwise.

func wqo(PN1:PN, s1:M, s2:M)
    return forall p in PN[0] | s1[p] <= s2[p];
end func;

Fig. 5. Well-quasi-ordering function described as inclusion ordering in WSTSL

As it has been mentioned above in the Algorithms section, Backward Reachability Method requires effective algorithm for computation of pred-basis. The algorithm to compute it for Petri Net was suggested in [4]. How it is described in WSTSL is shown in Fig. 6.

func pred(PN1:PN, K:set of M)
    var P1:P = PN1[0];
    var T1:T = PN1[1];
    var PT1:PT(P1,T1) = PN1[2];
    var TP1:TP(T1,P1) = PN1[3];
    var predecessors: set of M(P1) = { };
    for s in K
        for t in T
            if forall tp in TP1[t] | s[tp[1]] - 1 >= 0 then
                s1 = s;
                for pt in PT1[t]
                    s1[pt[1]] = s1[pt[1]] + 1;
                end for;
                for tp in TP1[t]
                    s1[tp[1]] = s1[tp[1]] - 1;
                end for;
                predecessors = predecessors with s1;
            end if;
        end for;
    end for;
    return predecessors;
end func;

func pb(PN1:PN, K:set of M)
    return min(pred(PN1, 1), wqo)
end func;

Fig. 6. Description of the pred-basis and pred functions in WSTSL

To solve the covering problem the initial state and the state which coverability it is required to check should be specified. Afterwards, backwardanalysis function should be invoked with appropriate arguments (Fig. 7).

var m0:M(P1) = {<"P1",1>,<"P2",0>,
    <"P3",2>,<"P4",1>};
var mc:M(P1) = {<"P1",1>,<"P2",0>,
    <"P3",1>,<"P4",2>};
backwardanalysis(PN1,wqo,pb,m0,mc);

Fig. 7. Description of the initial marking and the marking which coverability it is required to check with Backward Reachability Method invocation

The tool provides the user with the output that contains sequence of sets $K_i$, where $K_0 = \{m_c\}$, $K_{n+1} = pb(K_n)$, their union $\bigcup_{i \in \mathbb{N}} K_i$ and its minimal elements (basis). Finally, it is reported whether the analyzed state (marking) $m_c$ is covered or not (Fig. 8).
Fig. 8. Report of the tool for the backward analysis invocation

As it has been mentioned above in the Algorithms section, Finite Reachability Tree requires effective algorithm for computation of \( \text{Succ} \). How it is described in WSTSL is shown in Fig. 9.

To construct Finite Reachability Tree only the initial state should be specified. Afterwards, \textit{forwardanalysis} function should be invoked with appropriate arguments (Fig. 10).

```
func succ(PN1:PN, s:M)
    var F1:F = FN1[0];
    var T1:T = FN1[1];
    var PT1:PT(PN1,T1) = FN1[2];
    var TP1:TP(T1,P1) = FN1[3];
    var successors : set of M(P1) = { };
    for t in T
        if forall pt in PT1[t] | s[pt[1]] - 1 >= 0 then
            s1 = s;
            for pt in PT1[t]
                s1[pt[1]] = s1[pt[1]] - 1;
            end for;
            for tp in TP1[t]
                s1[tp[1]] = s1[tp[1]] + 1;
            end for;
            successors = successors with s1;
            end if;
        end for;
    return successors;
end func;
```

Fig. 9. Description of the \textit{Succ} function in WSTSL

The tool provides the user with the image which illustrates constructed Finite Reachability Tree (Fig. 11). Nodes are labeled with their states. Dead nodes are red. The node labeled with \{P1=1, P2=0, P3=2, P4=2\} state is dead since \{P1=1, P2=0, P3=2, P4=2\} \( \geq \) \{P1=1, P2=0, P3=2, P4=1\} (the latter state is represented by the root which subsumes the dead node labeled by the former state).

Fig. 10. Description of the initial marking and the Finite Reachability Tree construction invocation

Fig. 11. Constructed finite reachability tree
5.2. Lossy Channel Systems

Another model that we considered was Lossy channel system (LCS) which is a subclass of FIFO-channel systems.

**Definition 5.** FIFO-channel system is a 6-tuple \((S, s_0, A, C, M, \delta)\) where

- \(S\) is a finite set of control states;
- \(s_0 \in S\) is the initial control state;
- \(A\) is a finite set of actions;
- \(C\) is a finite set of channels;
- \(M\) is a finite set of messages (\(M^+\) is a set of finite strings composed of elements from \(M\));
- \(\delta\) is a finite set of transitions, each of which is represented by one of the following tuples \((s_1, c! m, s_2), (s_1, c? m, s_2), (s_1, a, s_2)\), where \(s_1, s_2 \in S\), \(c \in C\), \(m \in M\) and \(a \in A\) (see below).

Transition \((s_1, c! m, s_2)\) changes the control state from \(s_1\) to \(s_2\), adding the message \(m\) to the end of the channel \(c\). Operation \(c! m\) is also known as a send action.

Transition \((s_1, c? m, s_2)\) changes the control state from \(s_1\) to \(s_2\), removing the message \(m\) from the beginning of the channel \(c\). If the channel \(c\) is empty or its first element is not \(m\), then this transition cannot occur. Operation \(c? m\) is also known as a receive action.

Transition \((s_1, a, s_2)\) changes the control state from \(s_1\) to \(s_2\) and does not change the state of the channels.

Considering LCS it is also assumed that some message in some channel can be lost at any moment. To model this behavior one more operation \(\tau(c, m)\) is introduced.

Transition \((s_1, \tau(c, m), s_2)\) removes the message \(m\) from the channel \(c\), and does not change the control state.

For LCS \(= (S, s_0, A, C, M, \delta)\) the ordering \(\leq\) is defined on the set of global states \(\{(s, w)\} | s \in S, w : C \rightarrow M^+\) as follows:

\[(s, w) \leq (s', w') \iff s = s' \land w(c) \ll w'(c) \forall c \in C.\]

The ordering \(\ll\) is a subword ordering: \(u \ll v\) if \(u\) can be obtained by erasing letters from \(v\). It is shown in [6] that this ordering is a wqo.

The concrete model that we considered was Alternating Bit Protocol (ABP). It is represented by Sender and Receiver which communicate via two FIFO-channels \(c_M\) and \(c_A\). Sender sends messages to Receiver via \(c_M\), while Receiver sends acknowledgements via \(c_A\). Both channels can lose messages. Messages and acknowledgements contain one-bit sequence number 0 or 1. Sender continuously sends the same message with the same sequence number, until it receives an acknowledgement from Receiver with the same sequence number. Then, Sender changes (flips) the sequence number and proceeds with sending the next message. Receiver starts by waiting the message with the sequence number 0 (actually, it can initially send acknowledgments with the sequence number 1). When it receives such a message it starts sending acknowledgements with the same sequence number, until it receives the message with the flipped sequence number and so on. The described model is illustrated in terms of Lossy Channel System in Fig. 12.

![Fig. 12. Alternating Bit Protocol modelled as a Lossy Channel System](image_url)

5.3 Performance

To measure the performance of the implemented Finite Reachability Tree algorithm we applied it to the four different models, which include a model shown in Fig. 2 (Example 1) and the Petri Net models simulating the dining philosophers problem [10] for a number of philosophers equal to 5, 6 and 7. We executed the experiment on the following machine: Intel Core i7, 2.22 GHz, 16 GB RAM running OS X El Capitan (v. 10.11.6). System.nanoTime() method was invoked immediately before of the beginning of construction of a FRT and immediately after the end of construction, then the difference was calculated to measure run time for one run. In Table 1 in the Run time column average results for 20 runs are given in seconds. As well, sizes of the constructed FRTs are given. It can be seen that both run time and size of FRT grow exponentially for the philosophers problem.

<table>
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<th>Example</th>
<th>Run time (s)</th>
<th>Size of FRT</th>
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</tr>
<tr>
<td>Phil5</td>
<td>0.08587</td>
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</tr>
<tr>
<td>Phil6</td>
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6. Summary

This paper addresses a lack of practical results in studies of well-structured transition systems. In order to fill this gap, there was presented one of the possible ways for...
Development of the system capable to analyze WSTS with two common algorithms: backward reachability method and the Finite Reachability Tree. Well-Structured Transition Systems Language is introduced as a means of describing the user’s input, which consists of the description of transition system’s structure in general and specific instance’s relations and values.

The tool can be used by researchers to investigate the efficiency of the implemented algorithms. It is expected that it is appropriate for conducting experiments on small and medium-sized WSTS. The technology eases the efforts required to check the potential of the WSTS analysis algorithms for practical applications and to make what-if experiments on newly developed formalisms.

The application of the tool is illustrated for the Petri nets and Lossy Channel System formalisms. Also, there were given results of the experiment on Petri nets modeling the dining philosophers problem. The performance analysis of the Finite Reachability Tree applied to this problem demonstrated the expected exponential growth of execution time; and, it indicates the need for further investigations of optimizations (e.g. reduction rules) that can be applied to make the algorithm effectively applicable in practice.

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References

Инструмент для анализа поведения вполне структурированных систем переходов

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Аннотация. Вполне структурированные системы переходов являются хорошо известным инструментом для доказательства разрешимости свойств покрываемости и ограниченности. Каждый год появляются новые формализмы, которые оказываются вполне структурированными системами переходов. Несмотря на большой объем теоретической работы, существует большая потребность в эмпирических изучении вполне структурированных систем переходов. В данной работе представлен инструмент для анализа таких систем. Мы предлагаем расширение высокоуровневого языка SETL для описания вполне-структурных систем переходов. Это позволяет описывать новые формализмы близко к их формальному определению. Таким образом упрощается создание и изменение новых формализмов, а также осуществление анализа поведенческих свойств более сложных систем. Разработанный инструмент может быть полезным при внедрении и тестировании методов анализа формализмов, которые оказываются вполне структурированными системами переходов.

Ключевые слова: формальная верификация; системы с бесконечным числом состояний; вполне структурированные системы Переходов; сети Петри.

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