Two-dimensional direct numerical simulation of internal gravity wave attractor in trapezoidal domain with oscillating vertical wall

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Outline

Definition (Gravity waves)

Gravity waves are waves generated in a fluid medium with variable density or at the interface between two media when the force of gravity or buoyancy tries to restore equilibrium.

Example

Effect of vertical temperature variations on the wind near the ground.

Example

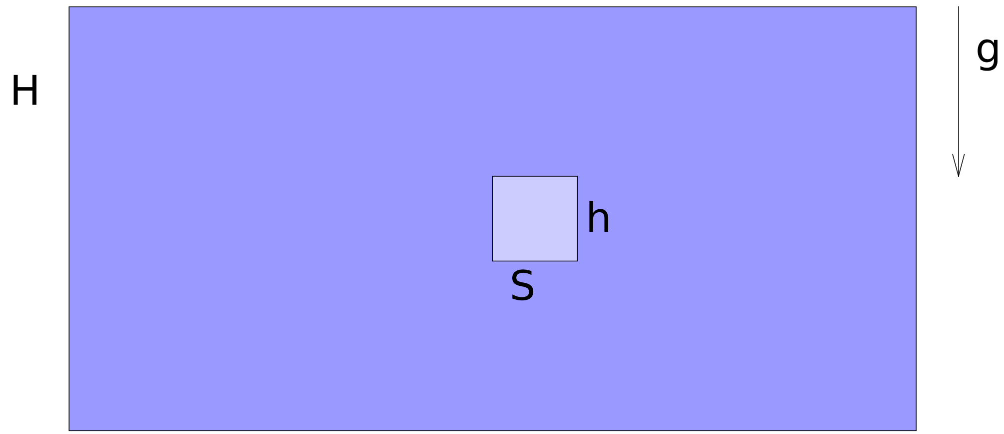
Effects of both temperature and salinity variations play an important role in many aspects of dynamical oceanography.

Forces acting on a fluid parcel in fluid subject to gravitation

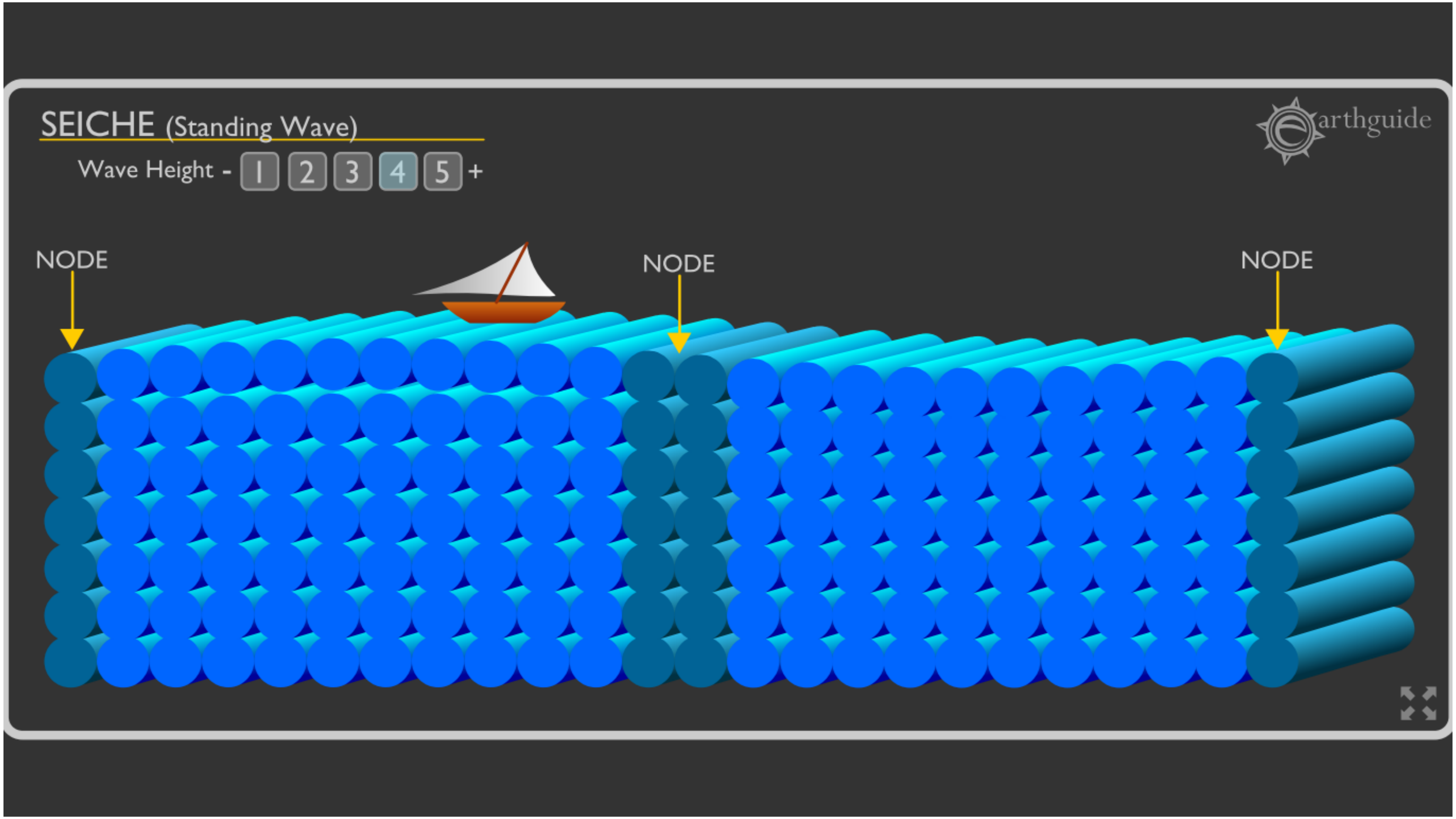
$$F = Sh\rho\vec{g}$$

$$P = (H - z)S\rho = pS$$

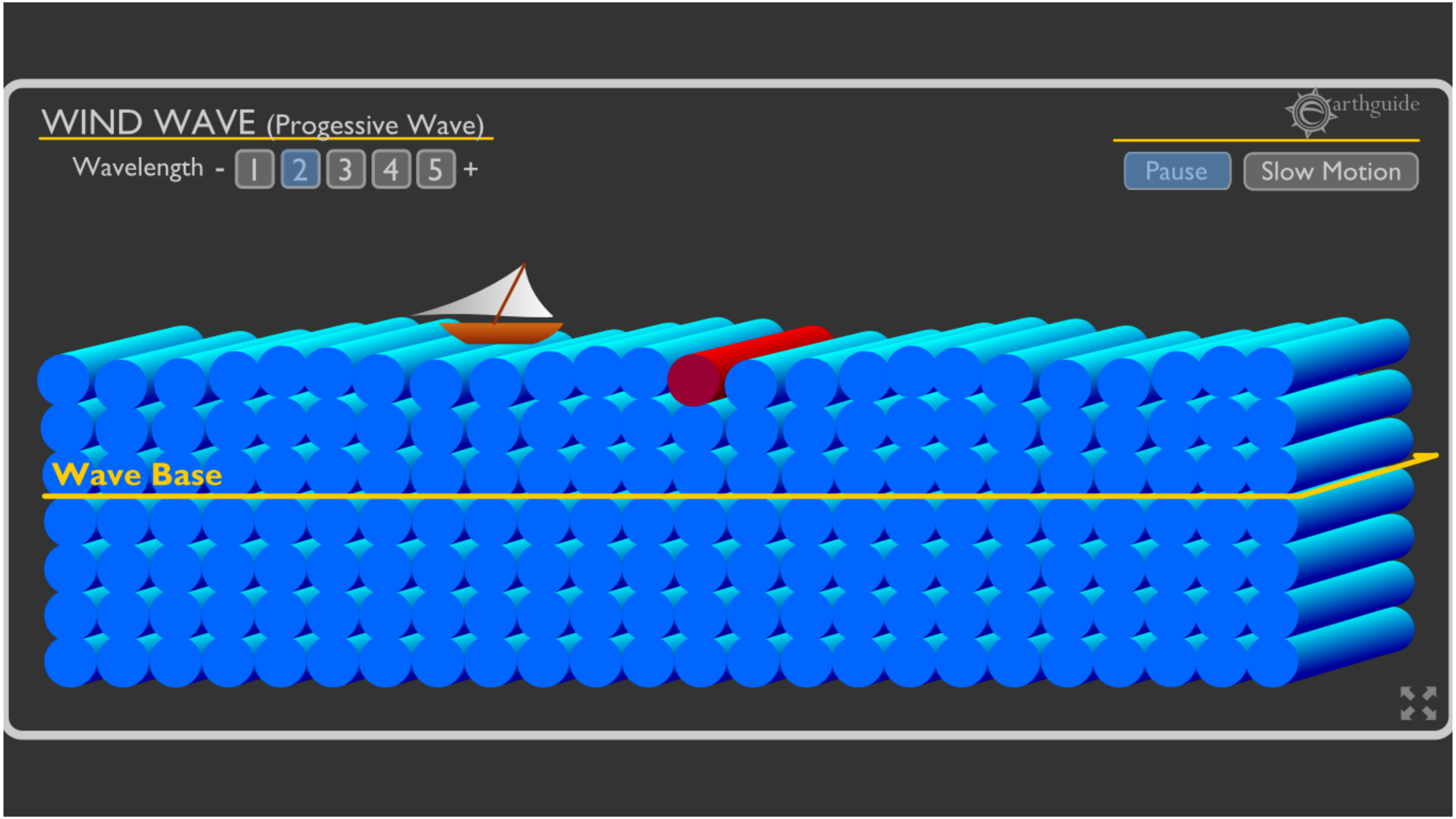
$$\frac{dp}{dz} = -\rho\vec{g}$$



Standing waves (seiche)



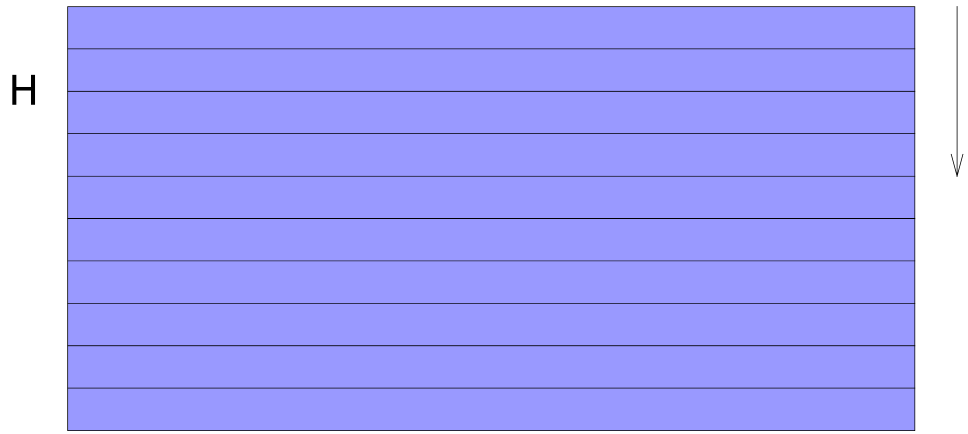
Traveling waves (ocean waves)



Stratified fluids

Stratified layer

Stratified layer consist of sublayers of constant density.
Such a layer is in *equilibrium*, but it may be unstable if density grows with height.



Stratified fluids

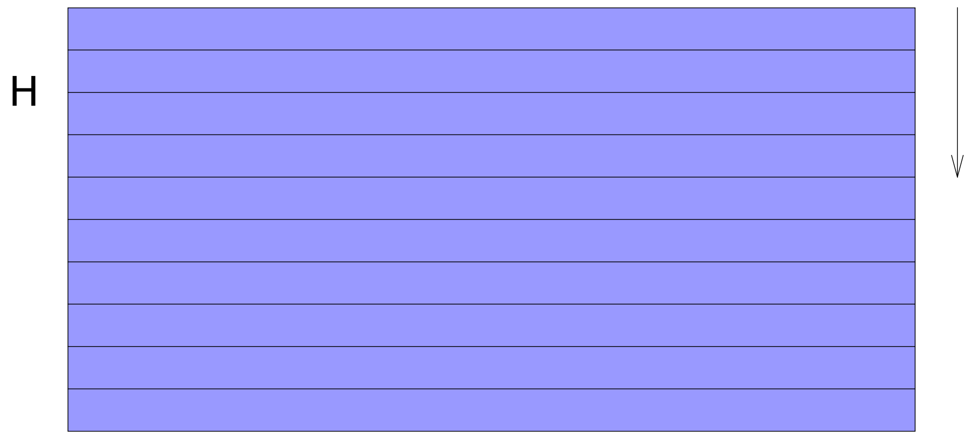
If fluid density decrease with height, fluid particle that does get displaced vertically tends to be restored to its original level; it may then overshoot inertially and oscillate about this level.

Let the unperturbed state of the layer has the density

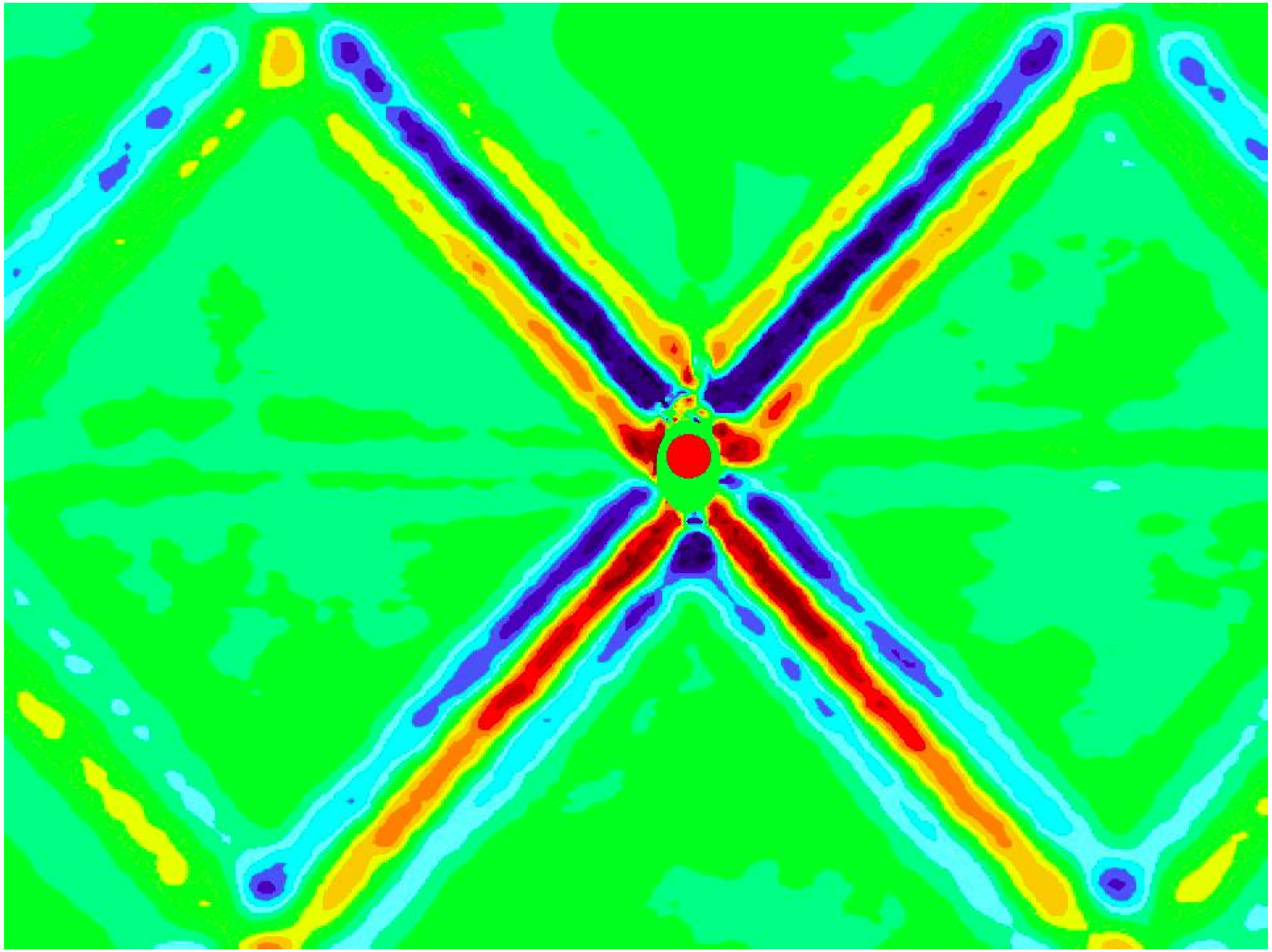
$$\rho(x, y, z, t) = \rho_m + \rho_0(z), \tag{1}$$

where ρ_m is the space average of the unperturbed state. Then the net gravitational force on a fluid particle after vertical displacement z is $-\frac{\mathrm{d}\rho_0}{\mathrm{d}z} z\vec{\mathrm{g}}$, so

$$\frac{\mathrm{d}^2 z}{\mathrm{d}t^2} = \frac{\mathrm{d}\rho_0}{\mathrm{d}z}\frac{\mathrm{g}}{\rho_0} z$$



Gravity waves from an oscillating cylinder



- Clear-Skies Turbulence
- Waves can travel up to 300 kilometers before breaking
- Thermocline in ocean, lakes (seasonal), atmospheric inversion

- If the density changes over a small vertical distance, the waves propagate horizontally like surface waves
- If the density changes continuously, the waves can propagate vertically as well as horizontally through the fluid.
- If moving vertically through the atmosphere where substantial changes in air density influences their dynamics, they are called anelastic (internal) waves.
- If generated in the ocean by tidal flow over submarine ridges or the continental shelf, they are called internal tides.
- If they evolve slowly compared to the Earth's rotational frequency so that their dynamics are influence by the Coriolis effect, they are called inertial gravity waves or, simply, inertial waves.

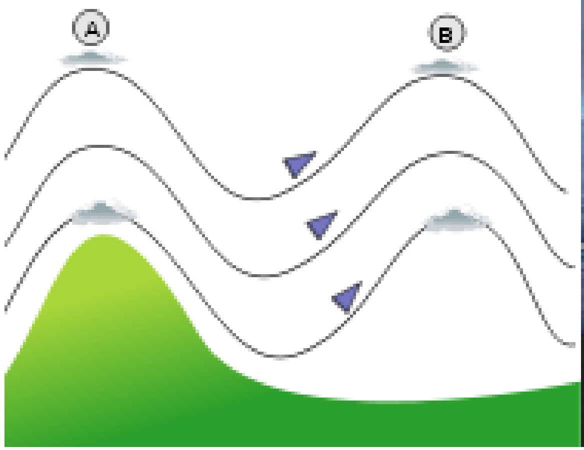
Seiche

Definition (Seiche)

Standing wave in an enclosed or partially enclosed body of water

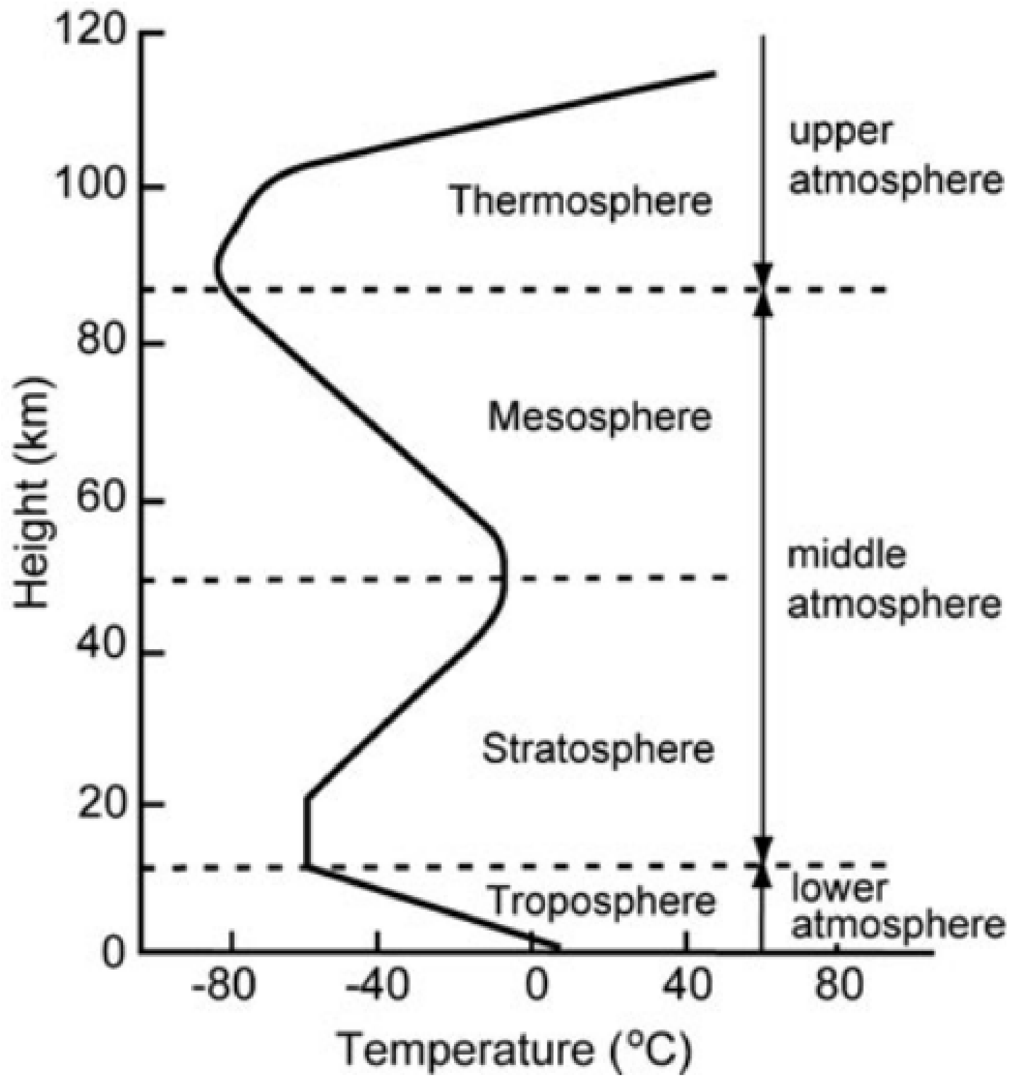
Definition (Lee waves)

Atmospheric standing waves caused by vertical displacement, for example orographic lift when the wind blows over a mountain or mountain range




Internal gravity waves (IGW) in atmosphere

- Parametrization of resistance of internal waves accounts for momentum and energy transfer by gravitational waves, which ar generated in troposphere and disappear in stratosphere and mesosphere.
- Necessity of calculations of amplitudes, wave energy fluxes, turbulent viscosities, and accelerations of the mean flow caused by IGWs generated in th troposphere.



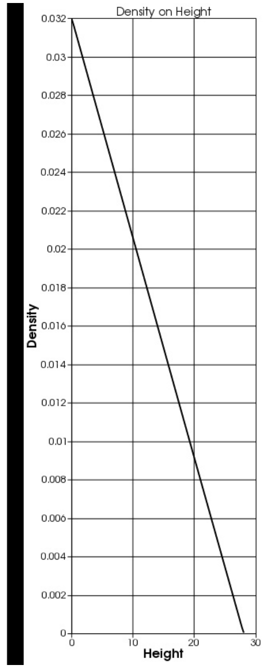
Importance of parametrization of momentum and energy depositions from gravity waves generated by tropospheric hydrodynamic sources for modeling of atmospheric circulation.

There are two types of parametrisation:

1. Orography resistance
2. Other sources, f.e. vertical shift of wind velocity or convection.

Usually it is assumed that gravity waves are generated on some level and spread upwards with momentum and energy.
A vertical heat flux through a unit area is written as $\vec{q} = \overline{p' w'}$, where p' – pressure perturbation due to gravity waves.
Two most known types of parametrizations are Hines Doppler-Spread and Warner and McIntyre Ultra-Simple Schemes

Stratified media in gravity field



Definition (Brunt-Väisälä frequency, or buoyancy frequency)

Brunt-Väisälä frequency, or buoyancy frequency, is the angular frequency at which a vertically displaced parcel will oscillate within a statically stable environment $\rho(z) = \rho_m + \rho_0(z)$:

$$\mathrm{N}(\mathrm{z}) = \sqrt{-\frac{\mathrm{g}}{\rho(z)} \frac{\mathrm{d}\rho(z)}{\mathrm{d}z}}$$

Conservation laws.

$$\rho\left(\frac{\partial \vec{\mathrm{v}}}{\partial t} + v^k \nabla_k \vec{v}\right) = -\nabla p + \mu \Delta \vec{v} + \rho \vec{g} + \frac{\mu}{3}\nabla(\nabla \cdot v)$$

$$\left(\frac{\partial s}{\partial t} + v^k \nabla_k s\right) = \lambda \Delta s$$

$$\frac{\partial \rho}{\partial t} + \mathrm{div}(\rho v) = 0$$

$$f(x, y, z, t) = f_m + f_0(z) + f'(x, y, z, t),$$

where f_m is the space average in absence of motion, $f_0(z)$ is the variation in absence of motion, f' is the fluctuation resulting from motion (notation of G. Veronis, 1963),
$s = (\rho - \rho_m)/\rho_m$, in this case ρ_m – minimal value of density.

Linearization in absence of viscosity and diffusivity.

In case of constant buoyancy frequency (which is not strictly equivalent to constant density gradient):

$$\rho \frac{\partial \vec{\mathrm{v}}}{\partial t} = -\nabla \mathrm{p} + \rho' \vec{g}$$

$$\frac{\partial \rho'}{\partial t} + \mathrm{w} \frac{\mathrm{d}\rho_0}{\mathrm{dz}} = 0$$

$$\mathrm{div}(\mathrm{v}) = 0$$

Wavelike solution, periodic in space and time

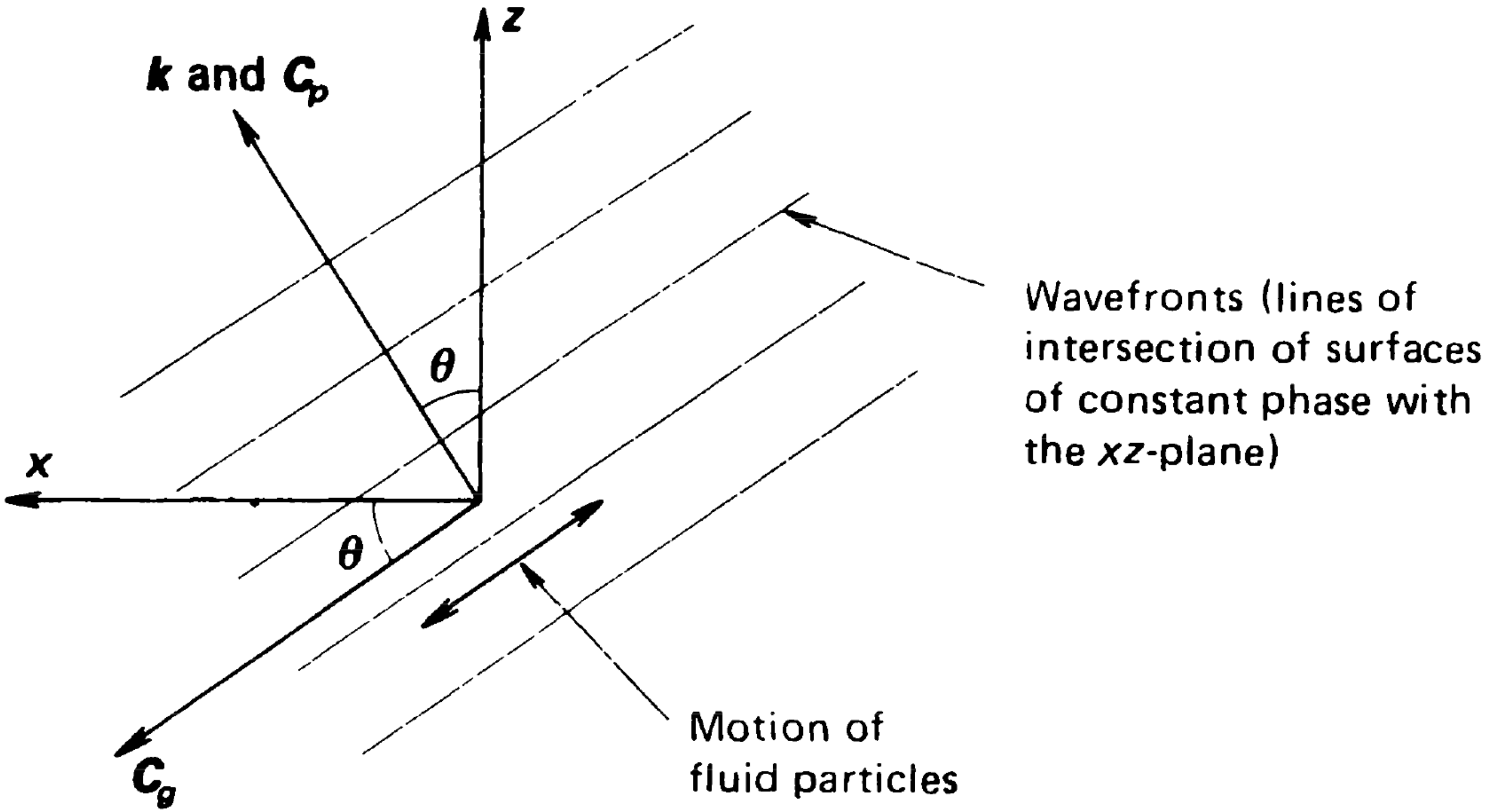
$$f = F \exp i(\omega t - (\vec{k}, \vec{r}))$$

$$\omega^2 = N^2 \left(1 - k_z^2 / k^2\right) = (N \left|\sin\theta\right|)^2,$$

where θ is the angle between the *vertical* axis and $\vec{k} = e_x k_x + e_y k_y + e_x k_z = e_x k \sin\theta + \ldots$ Waves exist for any value of the angular frequency from zero up to N.

Phase and group velocity

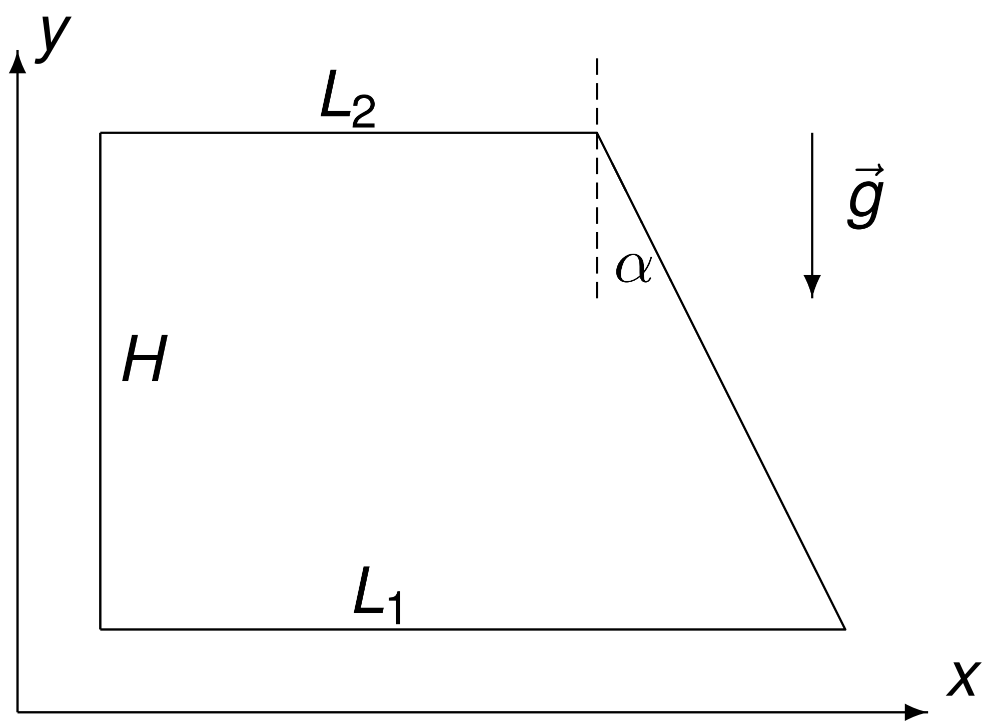
$$c_g = e_x \frac{\partial \omega}{\partial k_x} + e_y \frac{\partial \omega}{\partial k_y} + e_z \frac{\partial \omega}{\partial k_z} \tag{2}$$



System Of Equations In Boussinesq Approximation.

Geometry.

- A salt solution is confined in a trapezoidal domain.



Computational domain in 2D case.

- Initially constant vertical salt stratification is imposed with buoyancy frequency N.

System Of Equations In Boussinesq Approximation.

Boundary conditions.

- For constant viscosity and diffusion the equations are:

$$\left(\frac{\partial \vec{v}}{\partial t} + v^k \nabla_k \vec{v}\right) = -\nabla \frac{\tilde{p}}{\rho_0} + \nu \Delta \vec{v} + s\vec{g}$$

$$\frac{\partial s}{\partial t} + v^k \nabla_k s = \lambda \Delta s$$

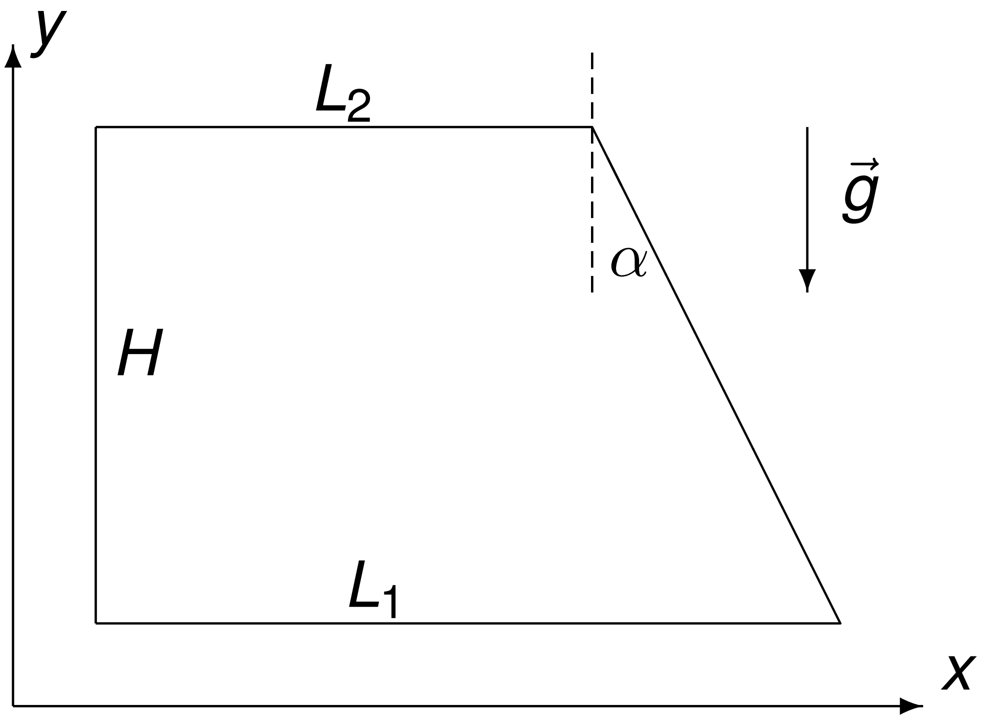
- A salt solution is confined in a trapezoidal domain.
  The upper boundary is stress free, the bottom and right boundary are rigid with no-slip condition, the left vertical boundary oscillate according to a given law.

$$x_b(0, y, t) = acos(\pi y/H)cos(\omega_0 t).$$

System Of Equations In Boussinesq Approximation.

Boundary conditions.

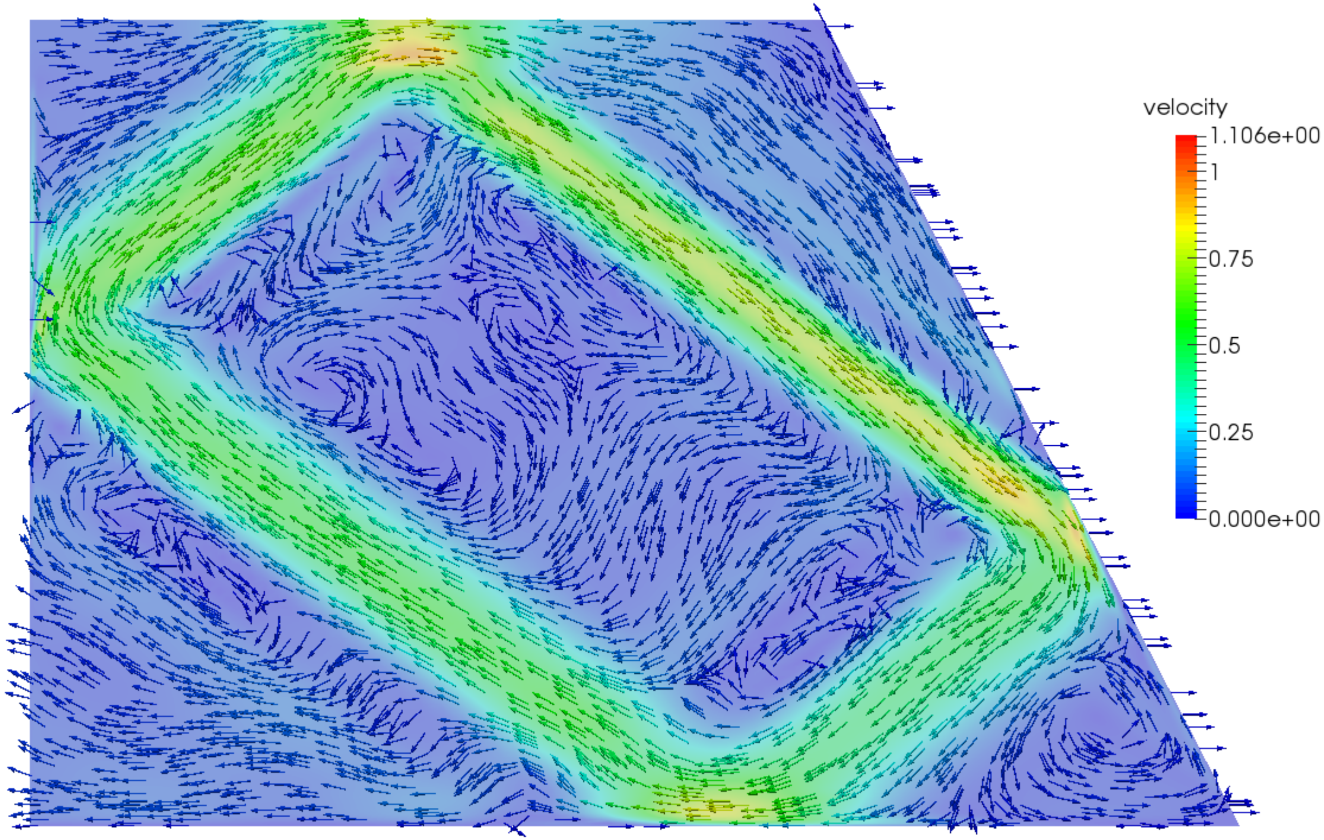
- On the bottom and right boundary $\{u = 0, v = 0\}$
- On the upper boundary $\{\partial u/\partial y = 0, v = 0\}$



- On the left boundary:

$$u(0, y, t) = a\omega_0 cos(\pi y/H) sin(\omega_0 t), v = 0.$$

Computational results. Velocity field.



Direction of velocity on the background of velocity magnitude computed for $a = 0.09\,cm$, $H = 28\,cm$, $N = 1.059 rad/s$ ($\omega_0 = 0.623 rad/s$)

Theoretical And Experimental Discovery Of Internal Wave Attractors

- Pioneer work Leo R. M. Maas & Frans-Peter A. Lam. Geometric focusing of internal waves. *J . Fluid Mech.*, 300:1–41, 1995. Existence of different types of internal wave attractors is predicted theoretically. A necessary condition for seiching to occur is formulated.
- First experimental results Maas L. R. M., Benielli D., Sommeria J., Lam F.-P. A. 1997. Observation of an internal wave attractor in a confined, stably stratified fluid. Nature 388, 557-561.

Further Experimental and Computational Researches of Internal Wave Attractors

- Laboratoire de Physique Ecole Normale Supérieure de Lyon Scolan, H., Ermanyuk, E., Dauxois, T. 2013. Nonlinear Fate of Internal Wave Attractors. Physical Review Letters 110, 234501.
- Maas L., Hazewinkel J., Tsimitri C., Dalziel S. 2010. Internal wave attractors in stratified fluids, robustness to perturbations. APS Division of Fluid Dynamics Meeting Abstracts L1049.
- Hazewinkel J., Tsimitri C., Maas L. R. M., Dalziel S. B. 2010. Observations on the robustness of internal wave attractors to perturbations. Physics of Fluids 22, 107102.
- MIT General Circulation Model Finite Volume Code. Grisouard, N., Staquet, C., Pairaud, I. 2008. Numerical simulation of a two-dimensional internal wave attractor. Journal of Fluid Mechanics 614, 1.
- Jouve, L., Ogilvie, G. I. 2014. Direct numerical simulations of an inertial wave attractor in linear and nonlinear regimes. Journal of Fluid Mechanics 745, 223-250.

For monochromatic waves of frequency ω

$$\Psi = \psi(x, z)e^{-i\omega t}$$

$$\frac{\partial^2\psi}{\partial^2 x} = \frac{\omega^2}{N^2}\frac{\partial^2\psi}{\partial^2 z}$$

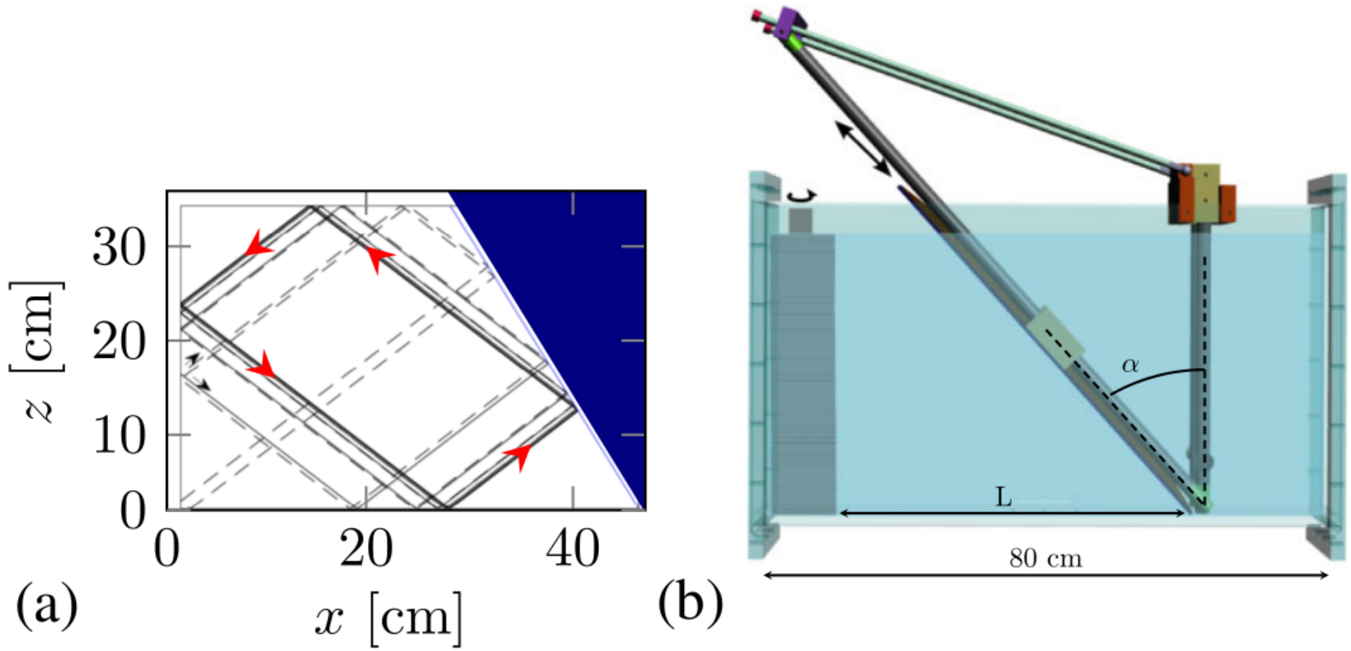
Dispersion relation:

$$\omega = \pm N\frac{k}{m} \tag{3}$$

k – horizontal wavenumber, m – vertical wavenumber.
With suitably re-scaled coordinates

$$\frac{\partial^2\psi}{\partial^2 x} = \frac{\partial^2\psi}{\partial^2 z}$$

Inner waves propagate at a fixed angle to the vertical that is determined solely by the forcing frequency.

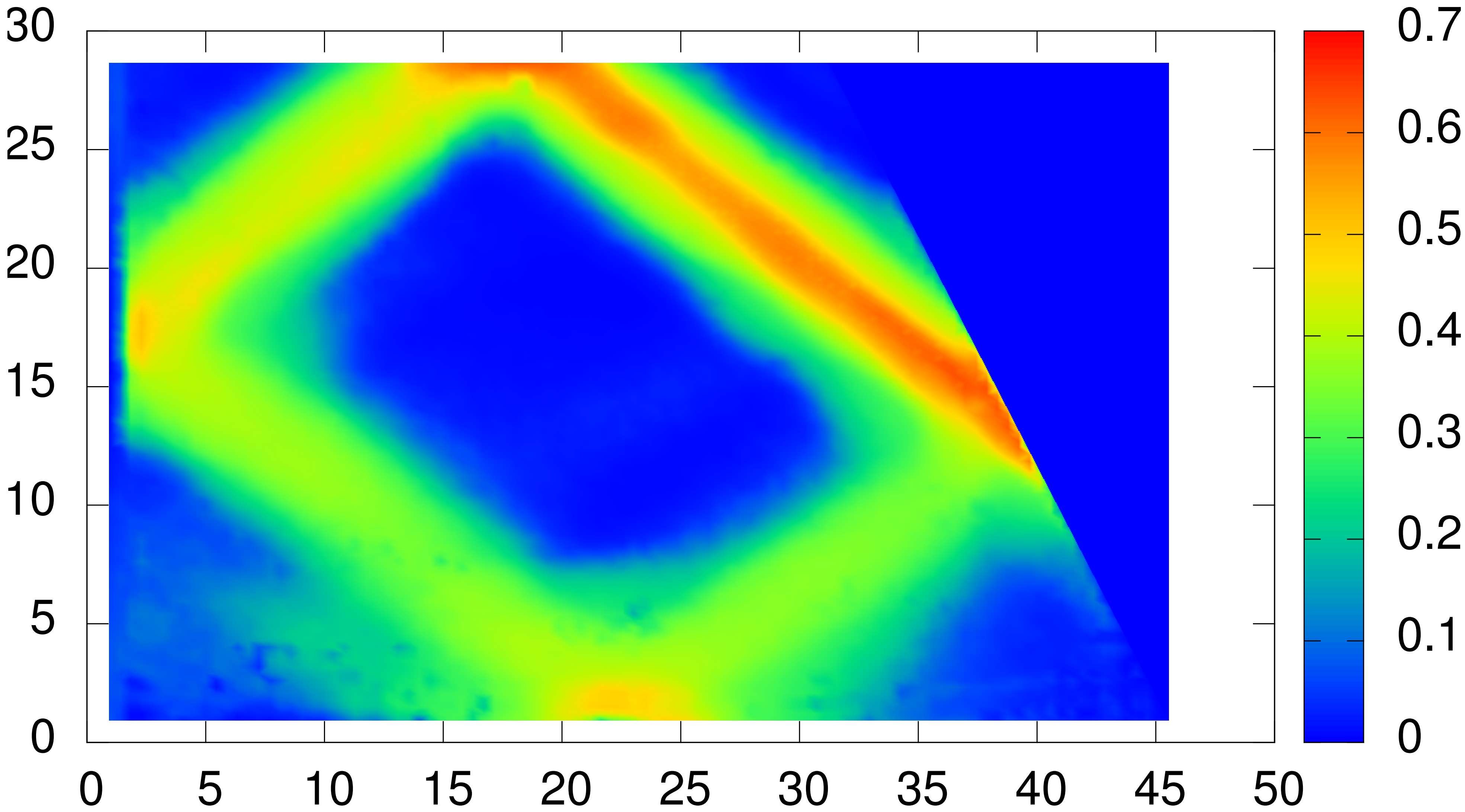


Scolan, H., Ermanyuk, E., Dauxois, T. 2013. Nonlinear Fate of Internal Wave Attractors. Physical Review Letters 110, 234501.

Set of parameters corresponding to the ongoing experiments (set II).

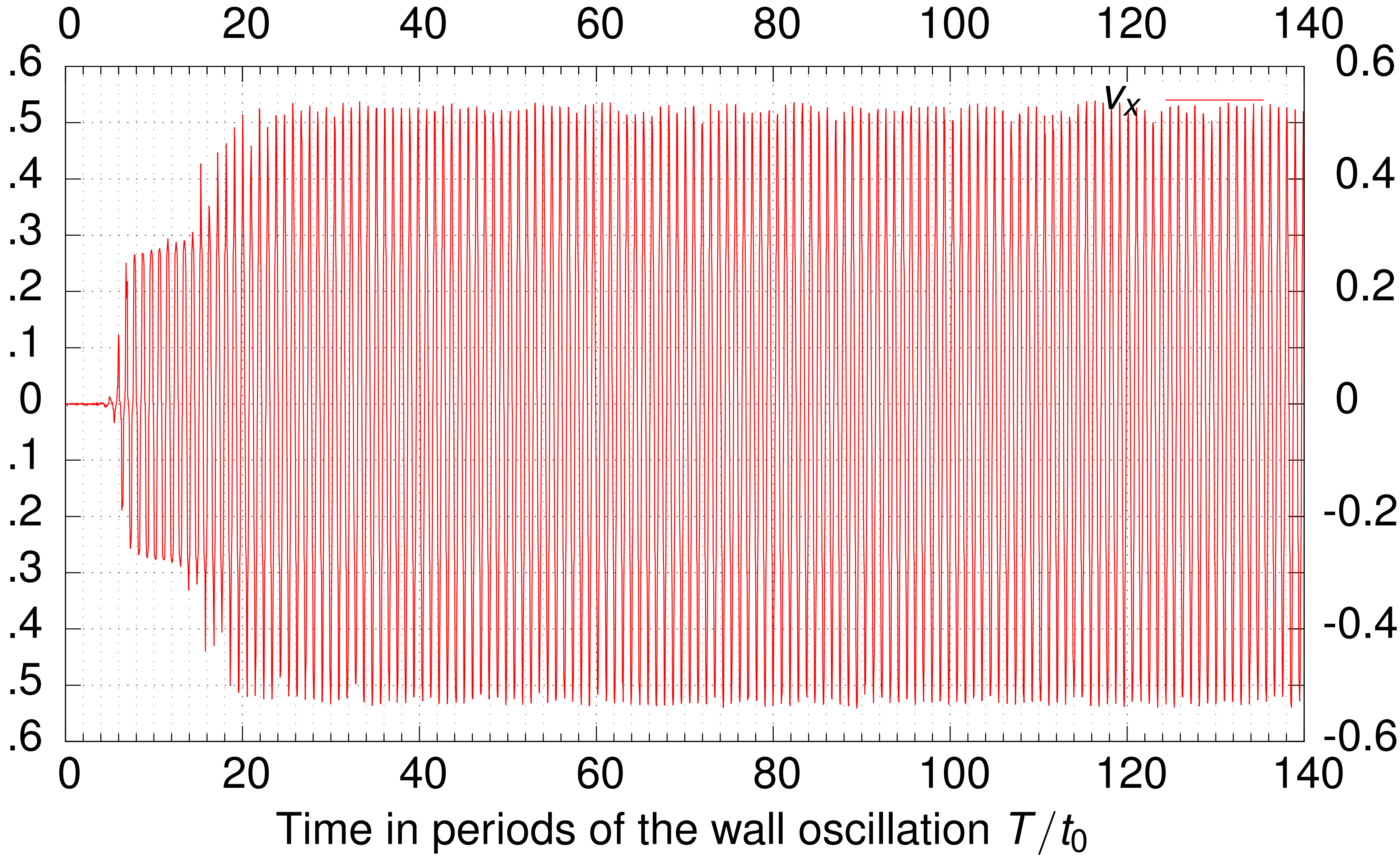
	SI	*CGS*
L_1	$0.45m$	$45cm$
L_2	$0.2951584m$	$29.51584cm$
H	$0.3m$	$30cm$
g	$9.81m/s^2$	$981cm/s^2$
ρ_0	$1000kg/m^3$	$1g/cm$
α	27.3^o	
N	$1.059rad/s$	$1.059rad/s$
ω_0	$0.623rad/s$	$0.623rad/s$
$d\rho_s/dz$	$-114.32kg/m^4$	$-0.0011432g/cm^4$
$\Delta\rho$	$34.296kg/m^3$	$0.034296g/cm^3$
ν	$10^{-6}m^2/s$	$10^{-2}cm^2/s$
κ	$10^{-9}m^2/s$	$10^{-5}cm^2/s$

Experimental results. Velocity amplitude field.



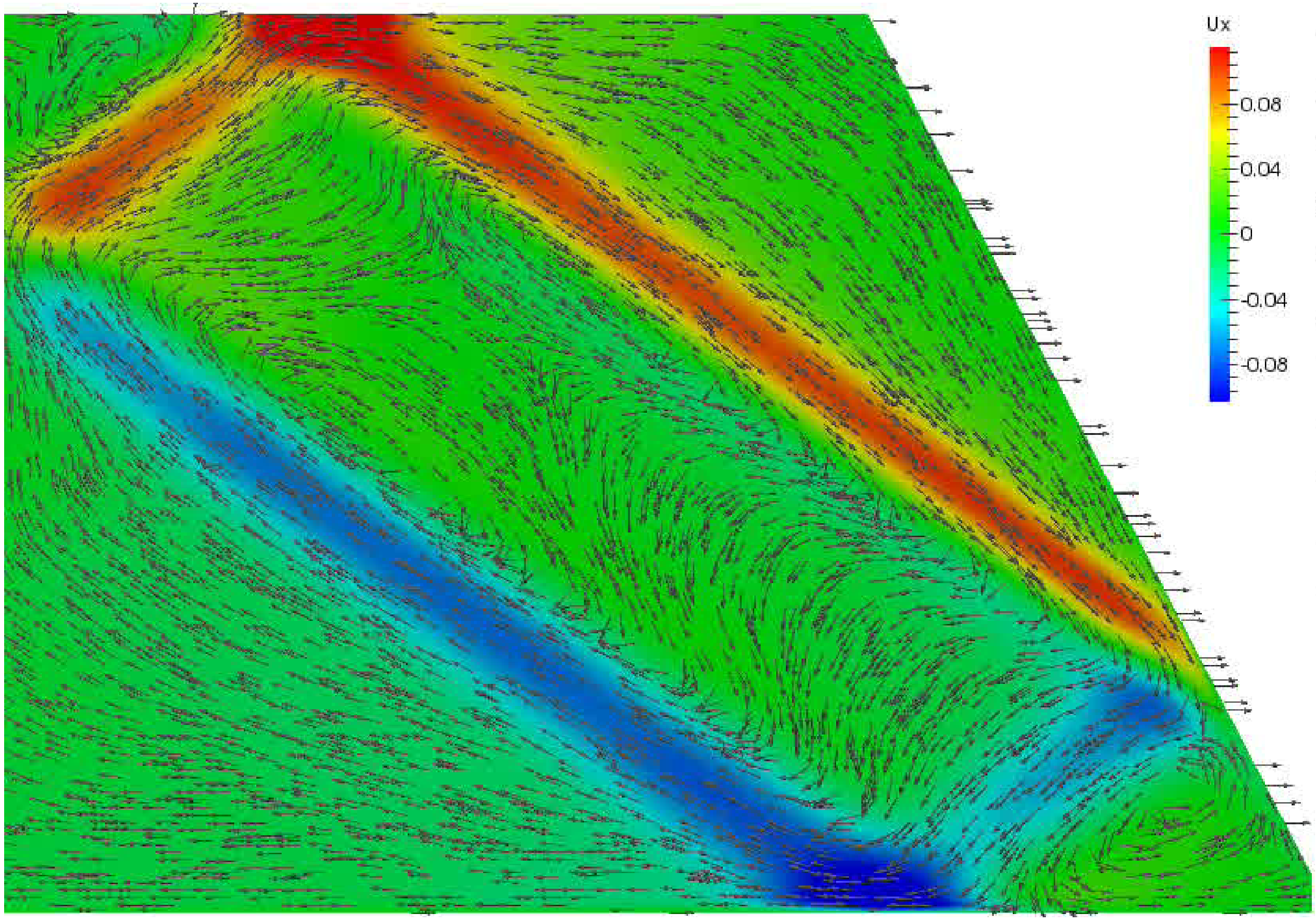
$a = 0.15cm, \omega_0 = 0.623rad/s$

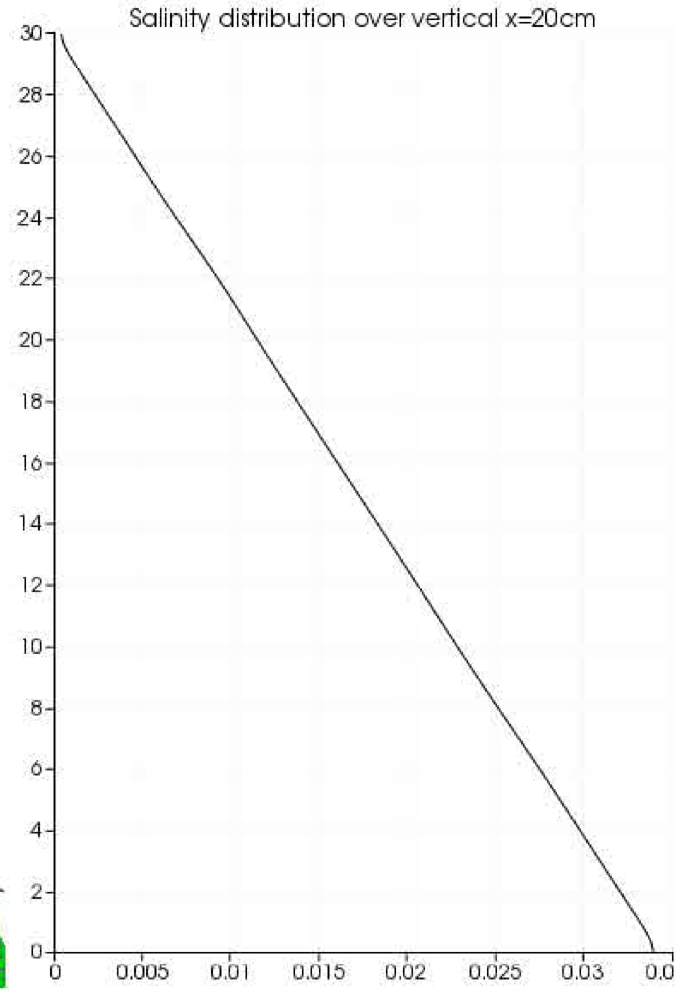
Experimental results. U_x at a point.



Computational results. Directions of velocity on the background of horizontal component of velocity.

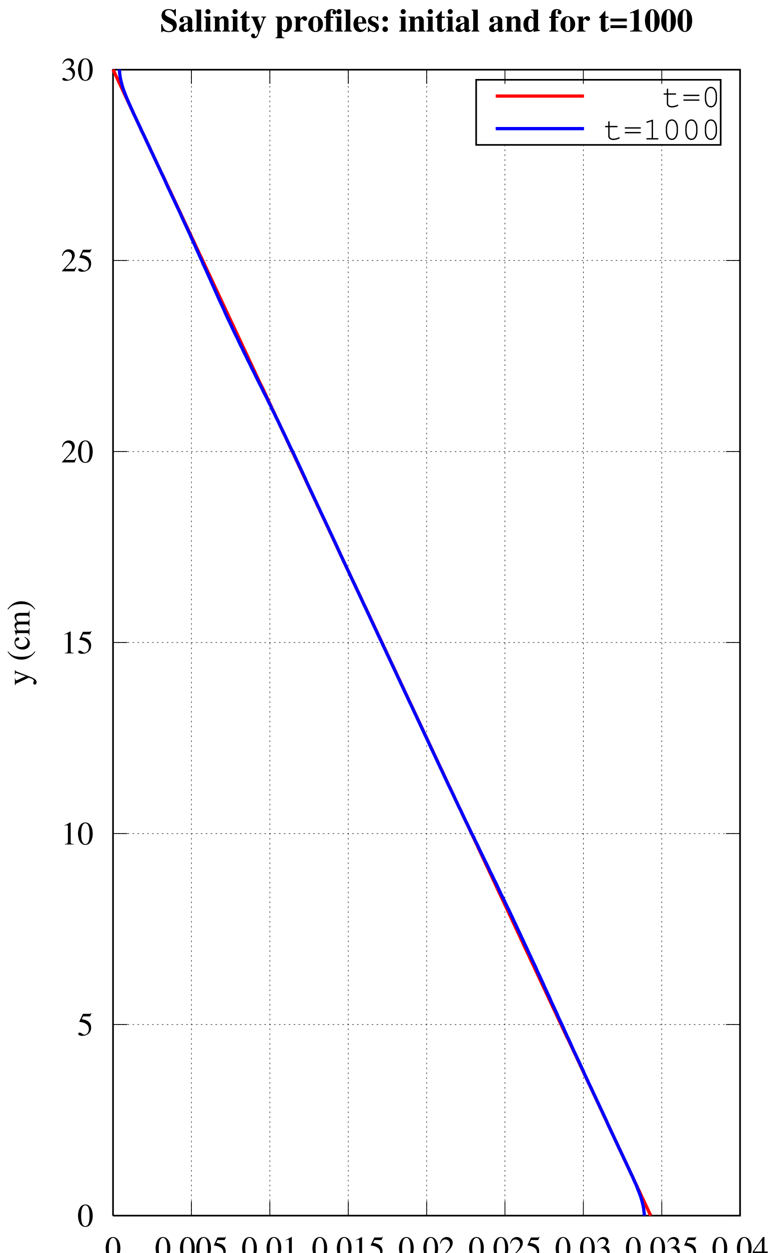
$N = 1.059 ras/s, \omega_0 = 0.623 rad/s, a = 0.02 cm, Pr = 100$

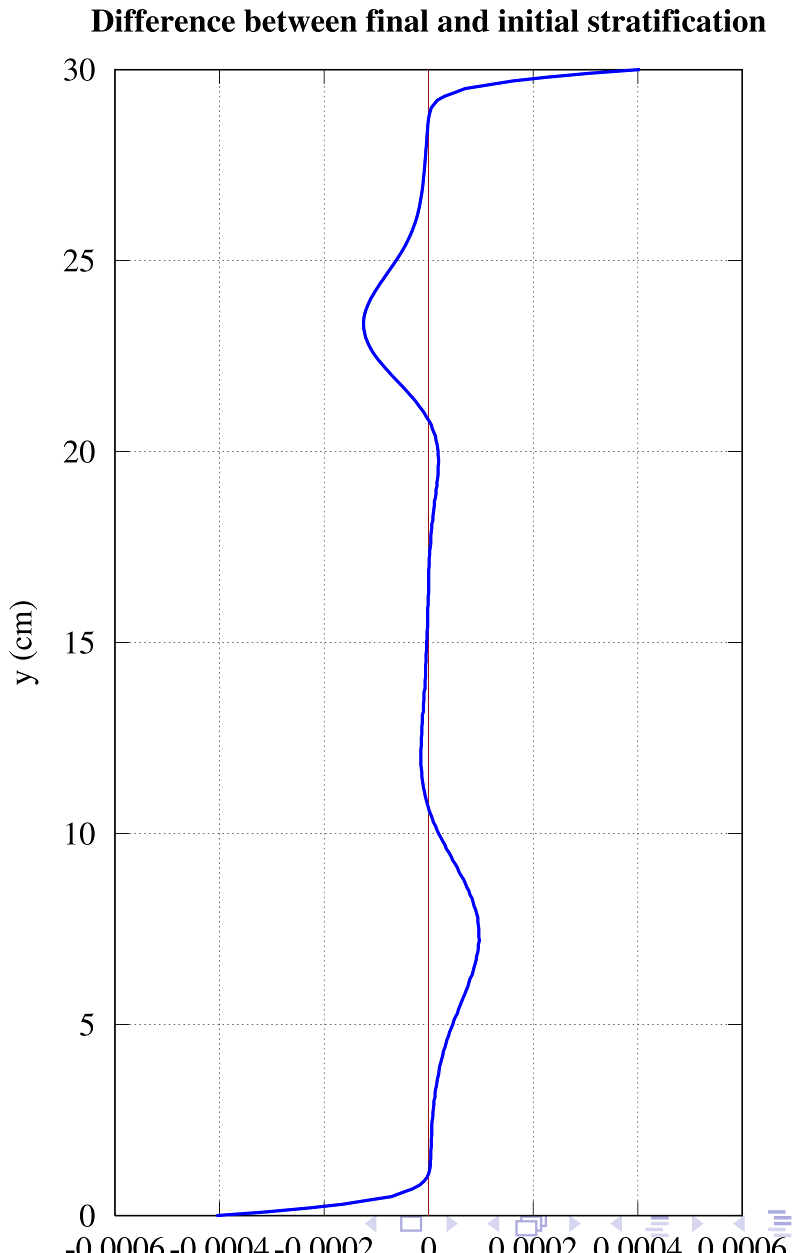




Computational results. Change of salinity profile with time.

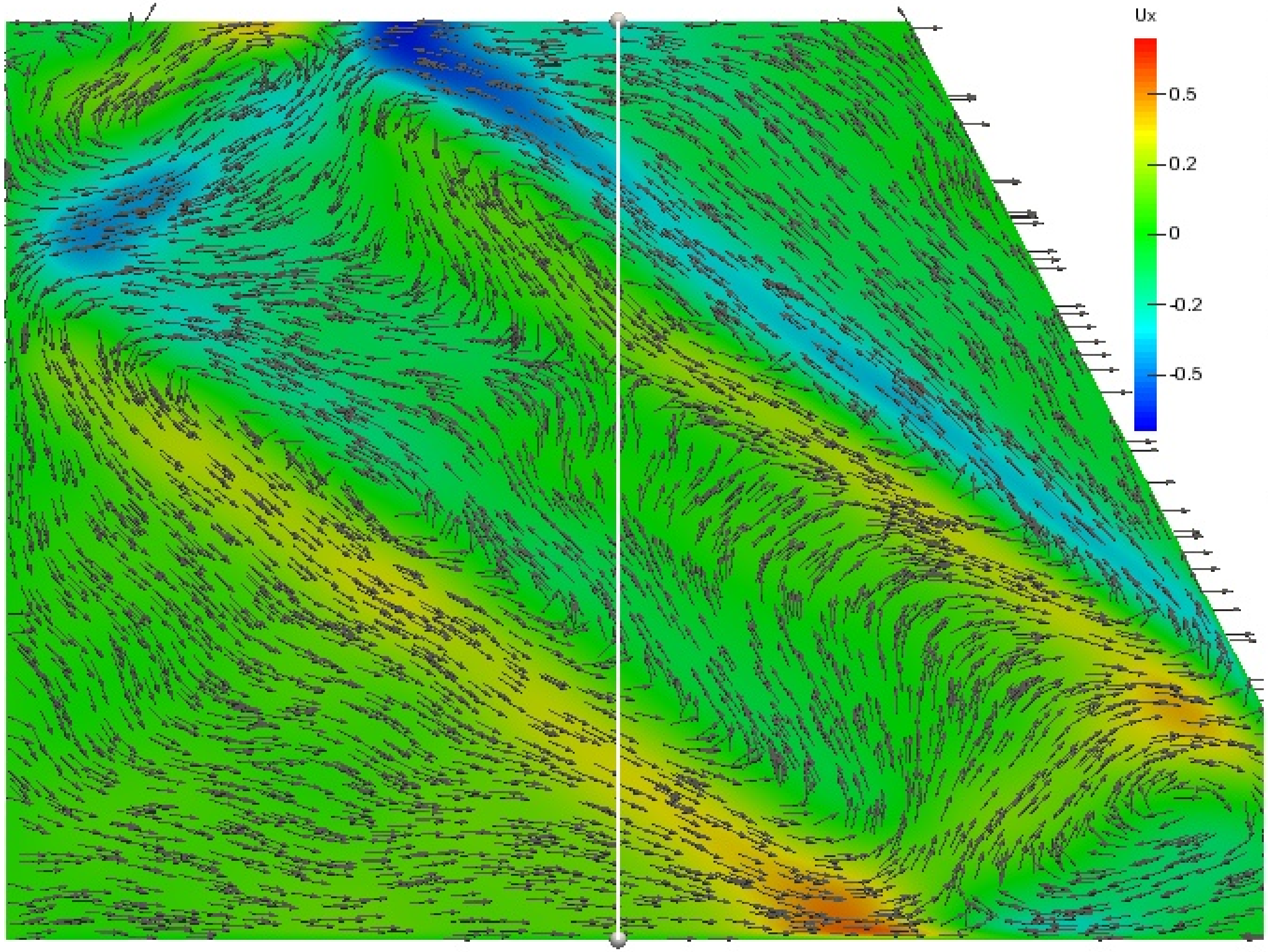
$N = 1.059 ras/s, \omega_0 = 0.623 rad/s, a = 0.02 cm, Pr = 100$ (7 times less than in computational model)

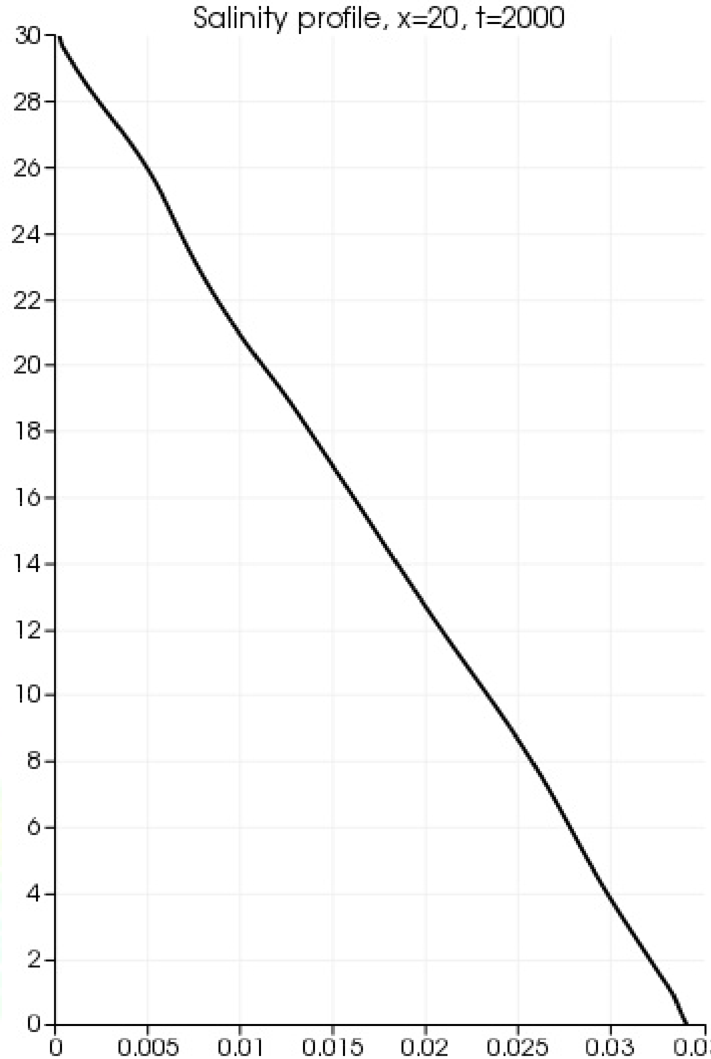




Computational results. Velocity direction on the background of horizontal component of velocity.

$N = 1.059 ras/s, \omega_0 = 0.623 rad/s, a = 0.105 cm, Pr = 700$





Computational results. Velocity direction on the background of horizontal component of velocity.

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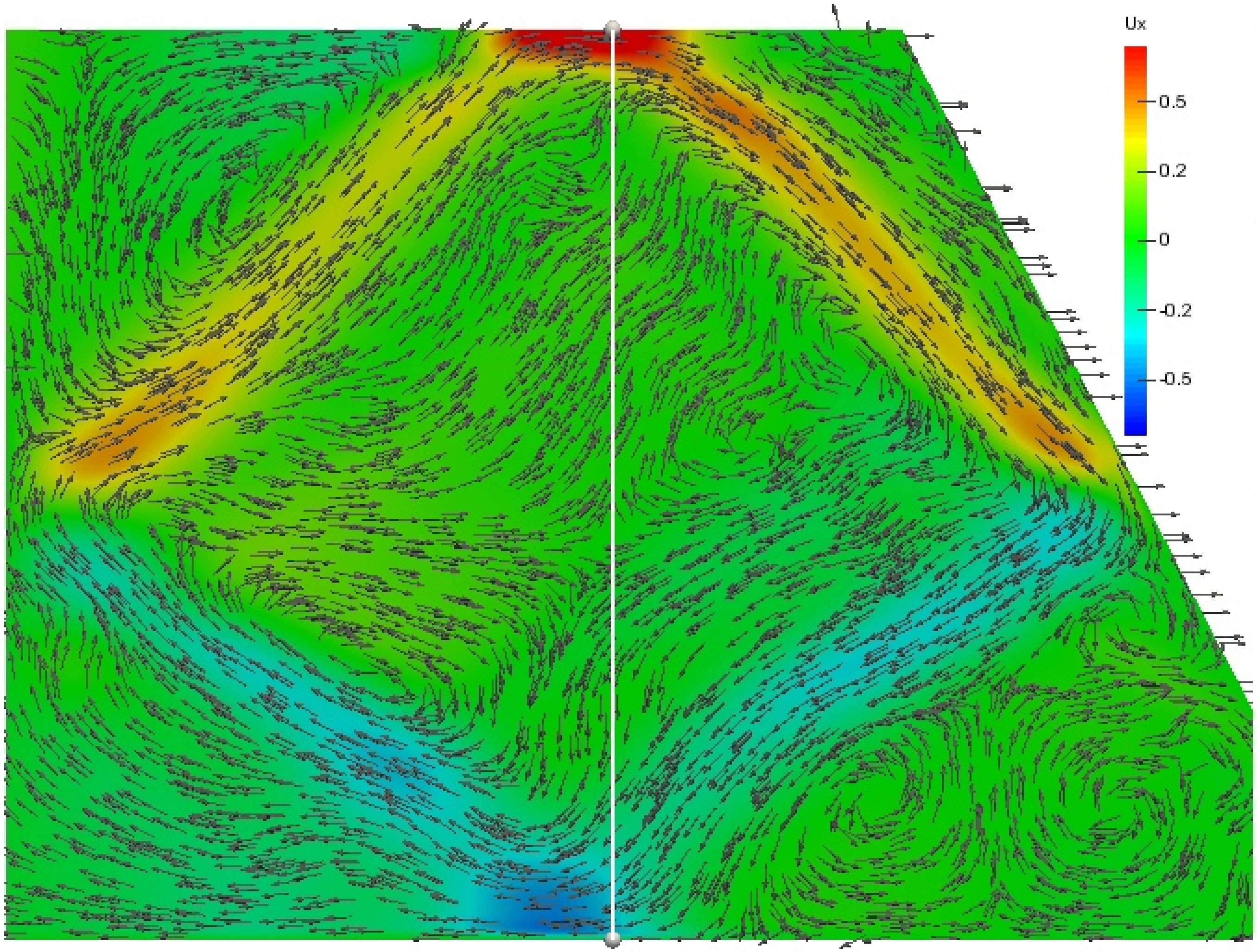
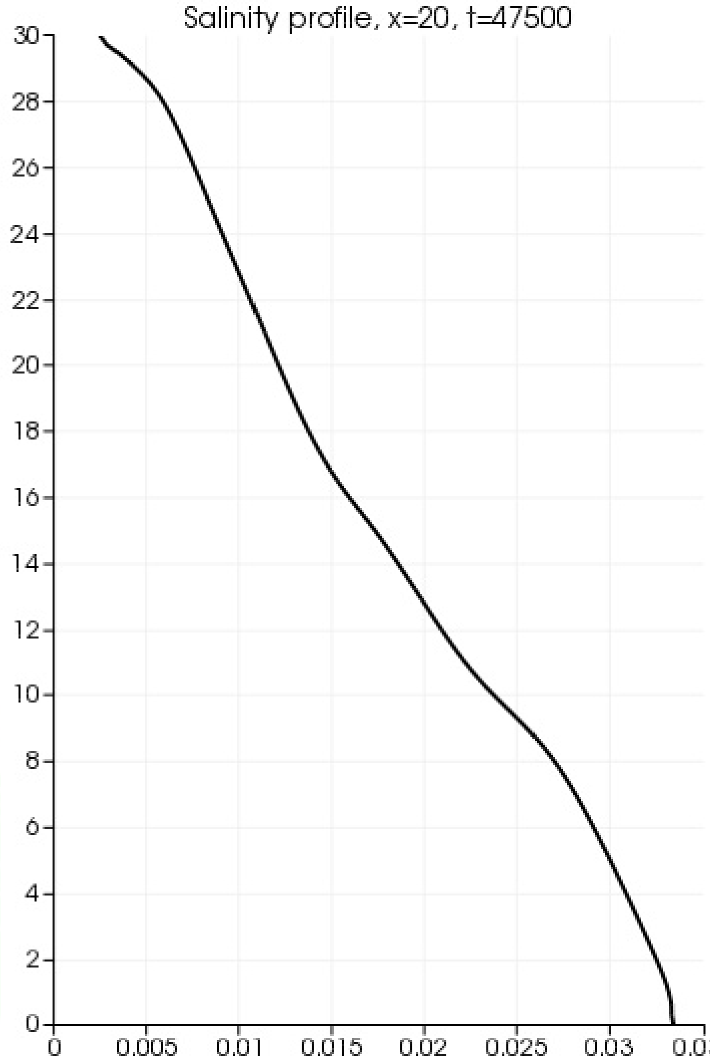
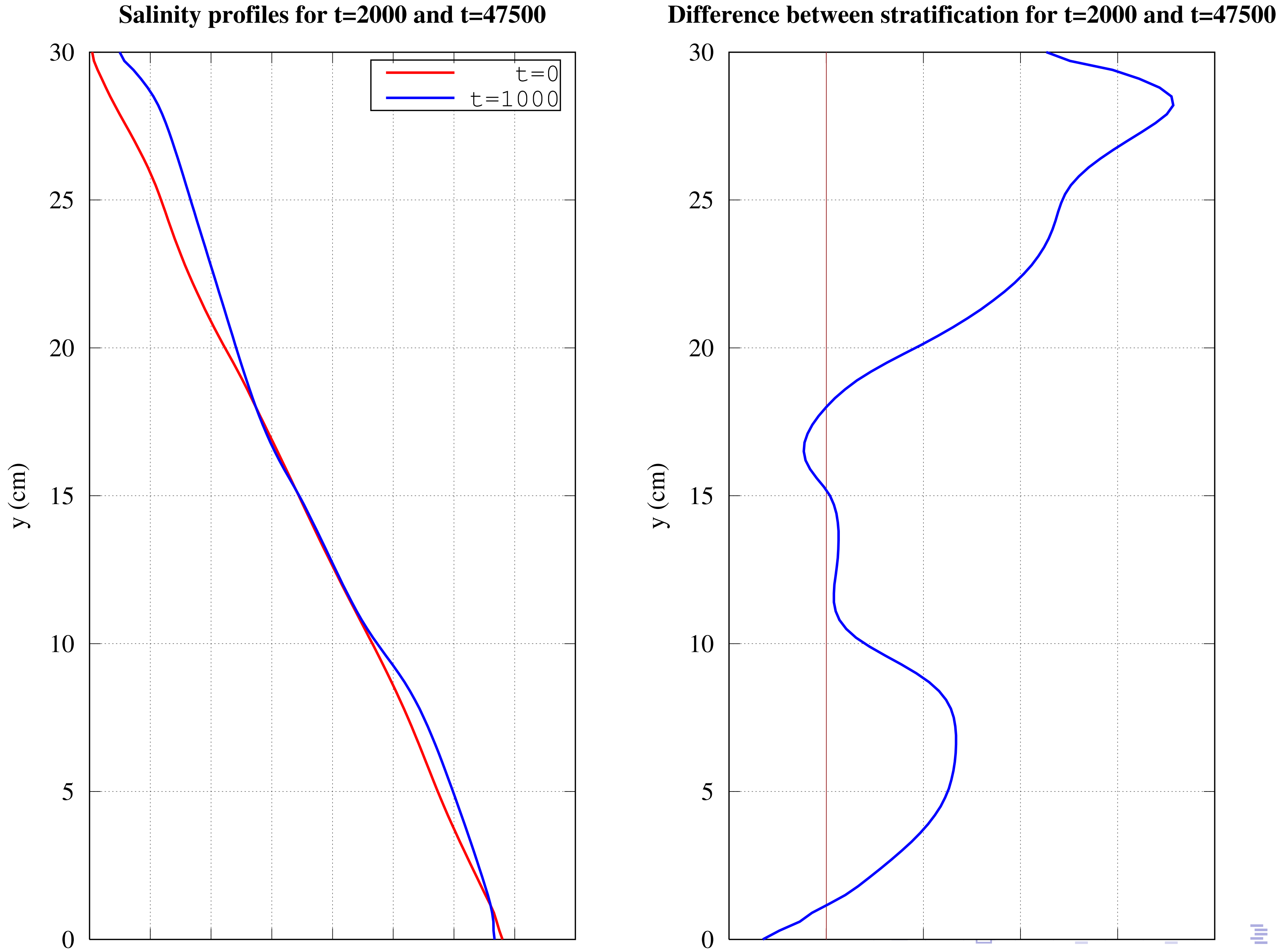


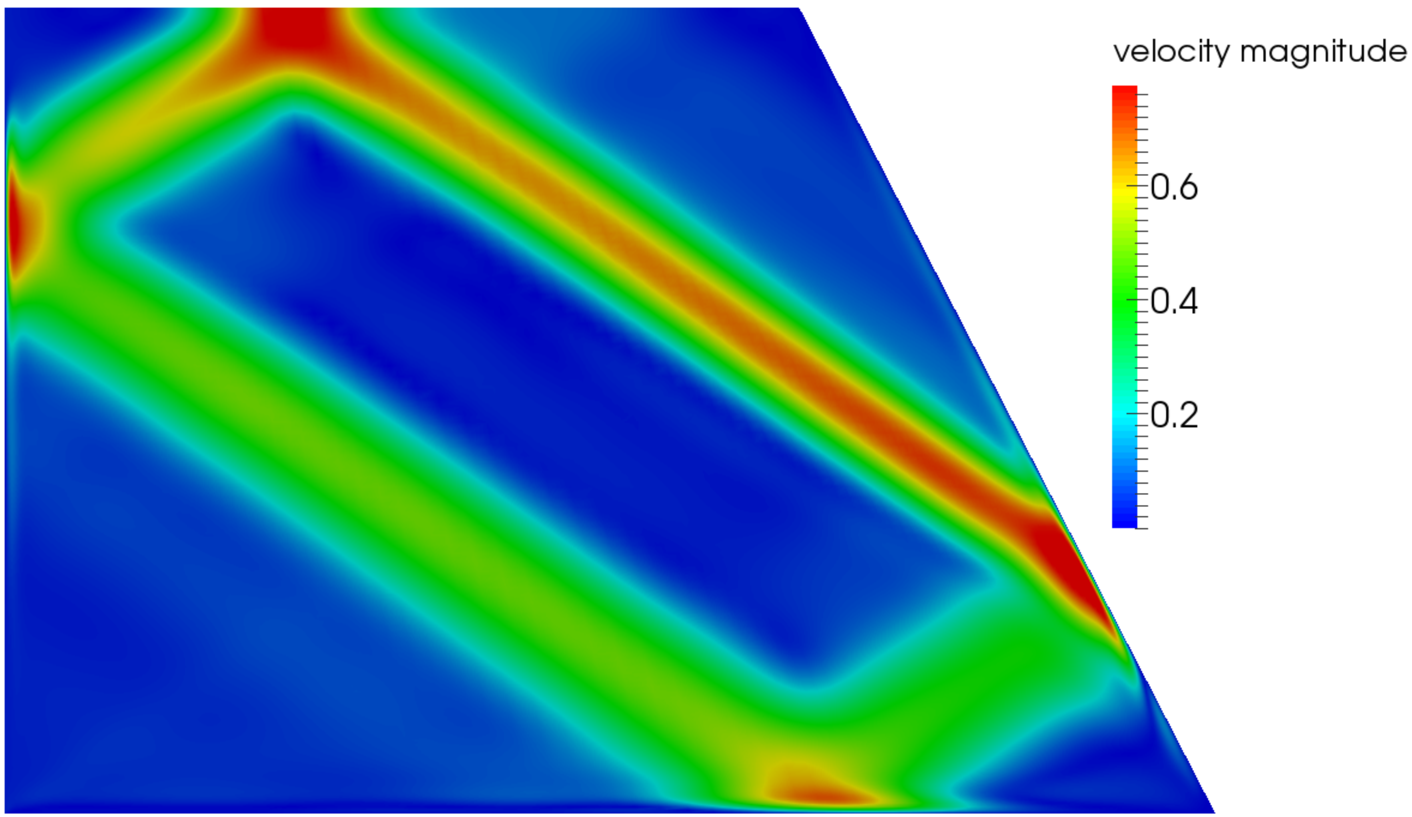


Computational results. Change of salinity profile with time.

$N = 1.059 ras/s, \omega_0 = 0.623 rad/s, a = 0.02 cm, Pr = 100$

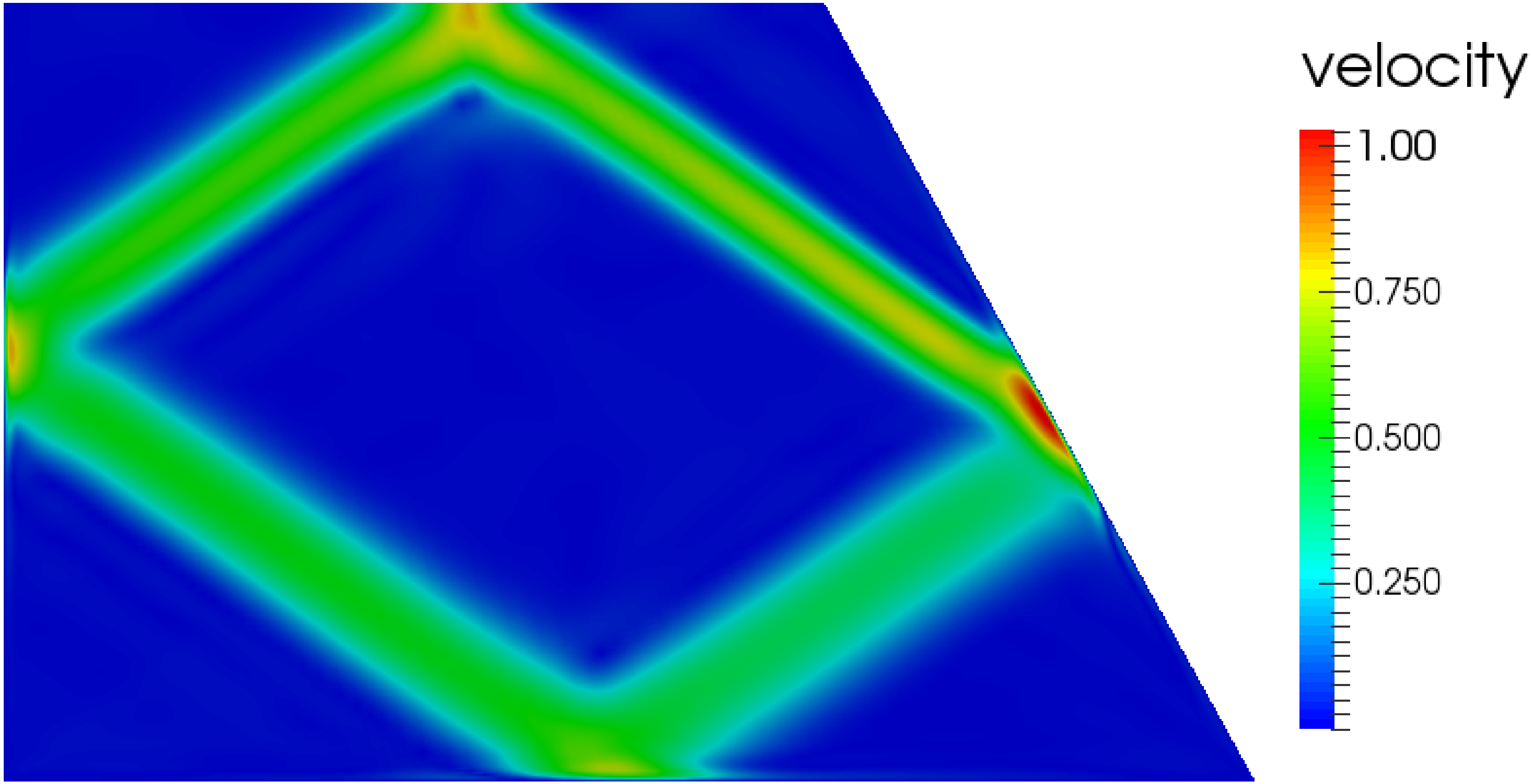


Computations using spectral elements. Magnitude of velocity field.



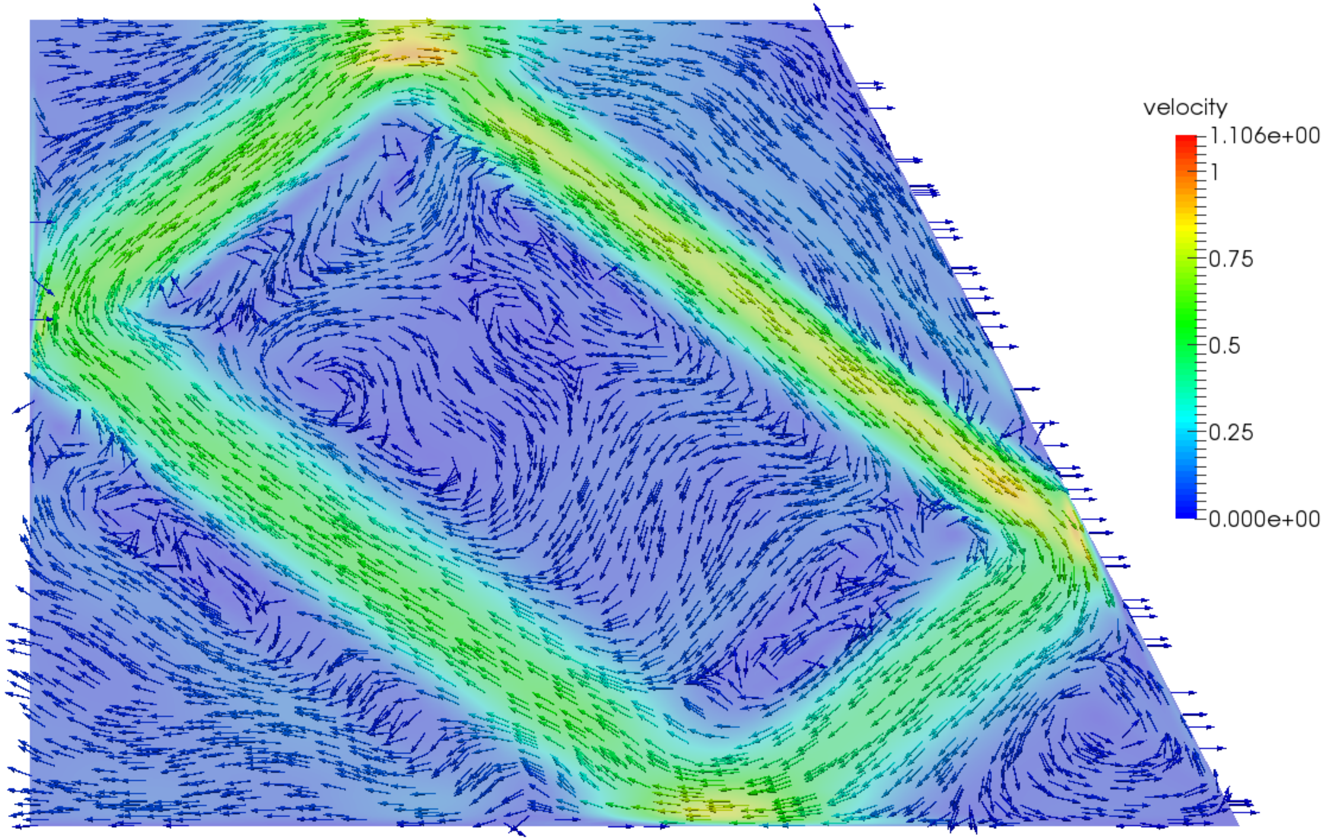
Account for imperfect stratification near boundaries

Computational results. Velocity magnitude field.



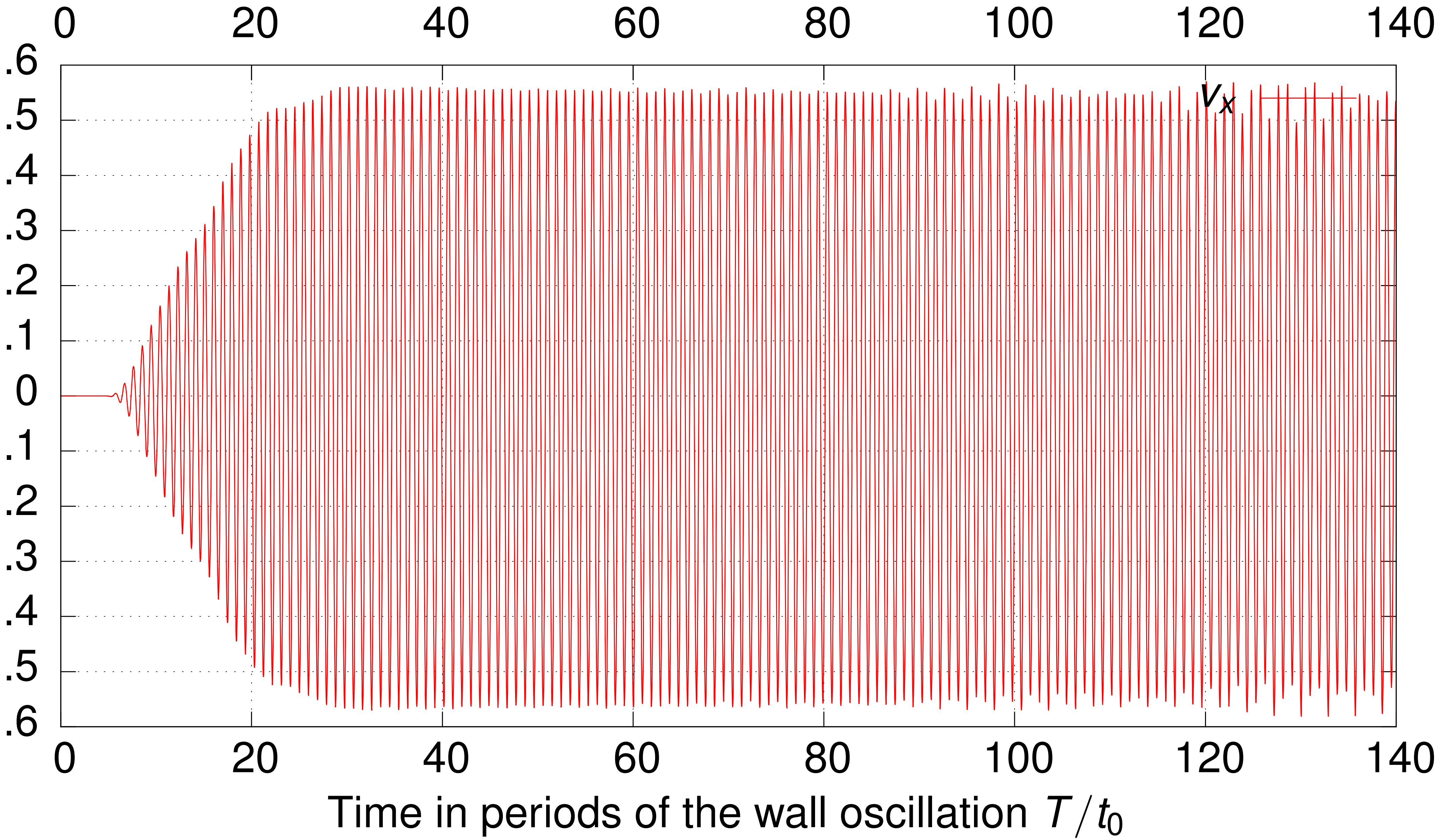
Velocity magnitude computed for $a = 0.09\,cm$, $H = 28\,cm$, $N = 1.059 rad/s$ ($\omega_0 = 0.623 rad/s$)

Computational results. Velocity field.

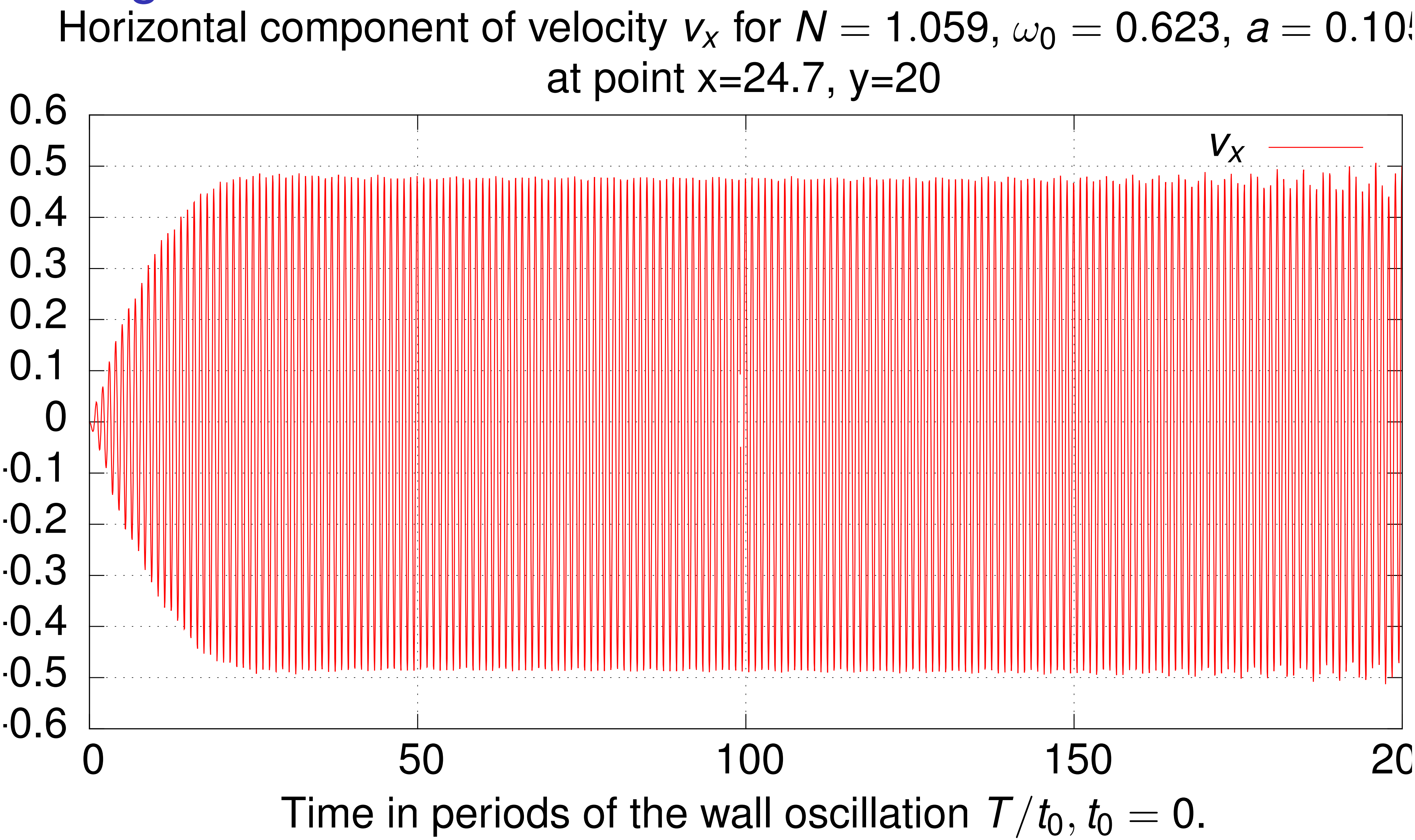


Direction of velocity on the background of velocity magnitude computed for $a = 0.09\,cm$, $H = 28\,cm$, $N = 1.059 rad/s$ ($\omega_0 = 0.623 rad/s$)

Computational results. U_x at a point. $a = 0.09cm$

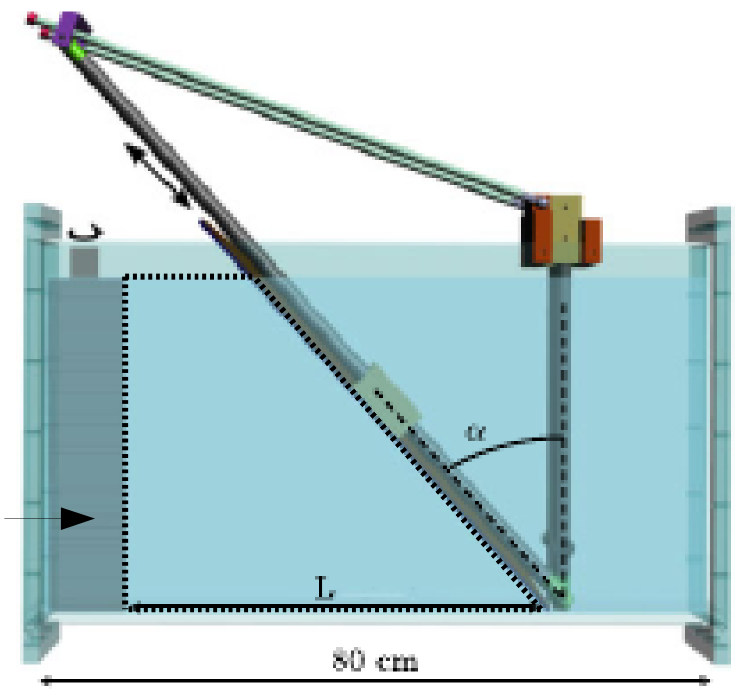


Long time behaviour



Конечно-объёмная модель стратифицированного течения в замкнутом пространстве

Описание проблемы и уравнения



Коленчатый вал, выдвигающий пластины в соответствии с заданным законом

Под солёностью здесь понимается отношение массы соли, растворённой в элементе жидкости к его общей массе (соль + вода)

$$s=\frac{m_s}{m_s+m_l}$$

Движение элементов жидкости описывается:

Уравнением неразрывности

$$\frac{\partial\rho}{\partial t}+\nabla\cdot\left(\rho\vec{U}\right)=0$$

Уравнением сохранения импульса

$$\frac{\partial\rho\vec{U}}{\partial t}+\nabla\cdot\left(\rho\vec{U}\otimes\vec{U}\right)-\nabla\cdot\sigma=-\nabla p+\rho\vec{g}$$

Уравнением сохранения массы растворённой соли

$$\frac{\partial\rho_s}{\partial t}+\nabla\cdot\left(\rho_s\vec{U}\right)=\nabla\cdot D_s\nabla\rho_s$$

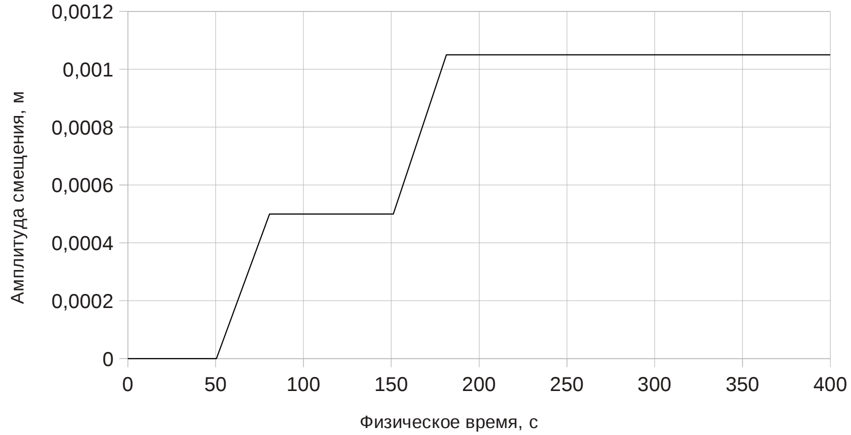
Уравнением «состояния», связывающим концентрацию растворённой соли и плотность жидкости

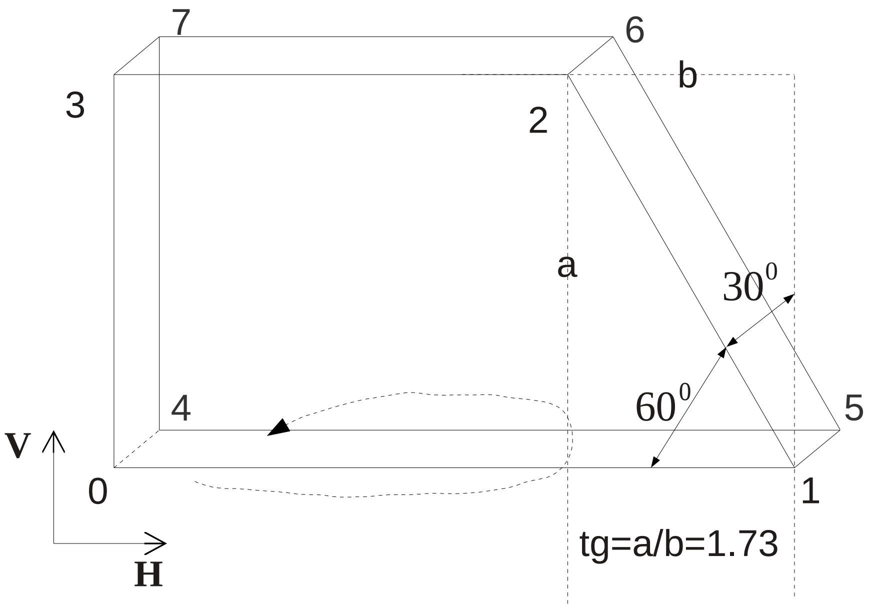
$$\rho(s)=\rho_0+\left(\frac{\partial\rho}{\partial s}\right)(s-s_0)$$

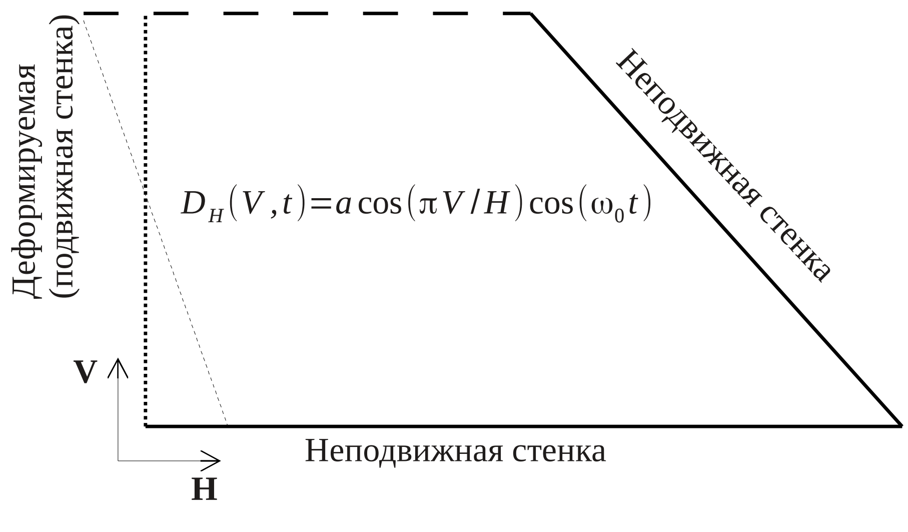
Расчётная область, граничные и начальные условия

Наклонная, нижняя и верхняя стенка неподвижны, вертикальная стенка «слева» колеблется в горизонтальном направлении по синусоидальному закону

В исследуемом режиме амплитуда колебаний увеличивалась от 0 до номинального значения примерно за 20 периодов







Приближение Буссинеска и особенности использования МКО

- Жидкость несжимаемая, Ньютоновская, принимается, что в балансе импульса градиент давления играет превалирующую роль по сравнению с изменением импульса частиц за счёт плотности ---> приближение Буссинеска

$$\nabla\cdot\left(\vec{U}\right)=0 \qquad \frac{\partial\vec{U}}{\partial t}+\nabla\cdot\left(\vec{U}\otimes\vec{U}\right)-\nabla\cdot\sigma=-\frac{\nabla p}{\rho_0}+\frac{\rho}{\rho_0}\vec{g}$$

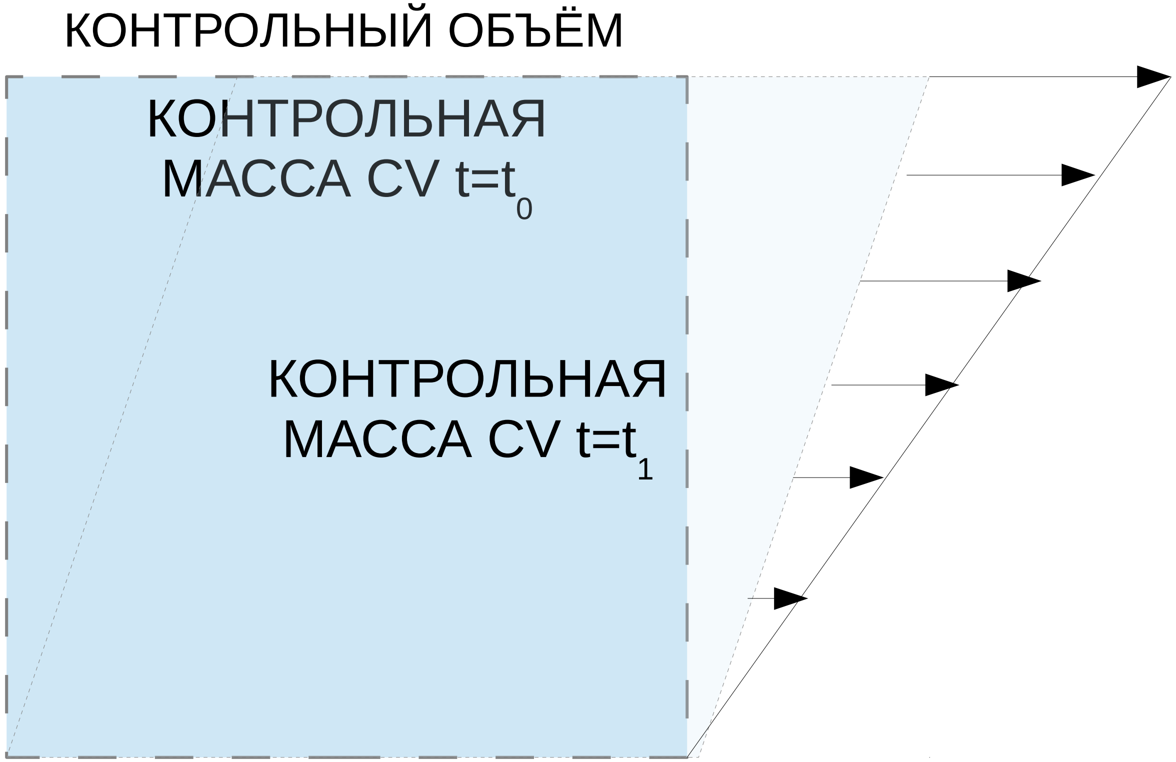
- МКО — метод численного решения **<u>интегральных</u>** уравнений! Все балансные соотношения должны быть сформулированы в интегральном виде:

$$\frac{d\Psi}{dt}=\frac{d}{dt}\int\limits_{CM(t)}\rho\psi\,dV=\frac{d}{dt}\int\limits_{CV(t)}\rho\psi\,dV+\int\limits_{\partial CV(t)}\rho\psi\left(\vec{U}_r\cdot d\vec{S}\right)$$

Или — изменение экстенсивного свойства «PSI» вычисляется как изменение интеграла этого свойства в заданном объёме плюс поток этого свойства через границы этого объёма.

<u>Все уравнения должны быть записаны в интегральной форме</u>

Подвижная и неподвижная сетки



Если решать задачу для замкнутой системы с использованием неподвижного контрольного объёма, то получится, что среда «покидает» или «заходит» через те его границы, которые не параллельны движению контрольной массы

Такую задачу лучше решать с использованием механизмов учёта подвижности контрольных объёмов

Численная реализация

1) Небаланс импульса на первом шаге

Градиент давления должен уравновешиваться столбом жидкости:

$$\nabla p = \rho \vec{g}$$

Необходимо либо инициализировать давление соответствующим образом, или <u>вычесть этот небаланс</u>

2) Уравнение для давления получаем с учётом небаланса масс на 0-ом шаге

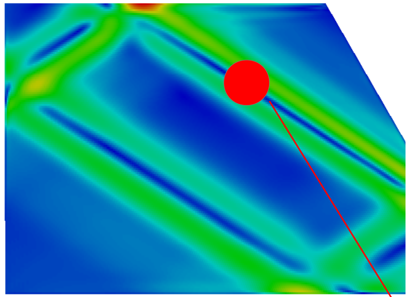
$$\sum_f \left(\frac{\langle H(\vec{U}_a) \rangle}{A} - \frac{\langle \vec{g}\cdot\vec{r}\,\nabla\rho_k - \vec{g}\cdot\vec{r}(t=0)\,\nabla\rho_k(t=0) \rangle}{A} \right)_f \cdot \vec{S}_f = \sum_f \left(\frac{\langle \nabla \frac{p^*}{\rho_0} \rangle}{A} \right)_f \cdot \vec{S}_f$$

3) Граничное условие для давления на стенках получаем

из условия равенства потоков 0

$$\int_{\partial CV(t)} \frac{p^*}{\rho_0} + (\vec{g}\cdot\vec{r})\rho_k\, d\vec{S} - \int_{\partial CV(t)} (\vec{g}\cdot\vec{r}(t=0))\rho_k(t=0)\, d\vec{S} = 0$$

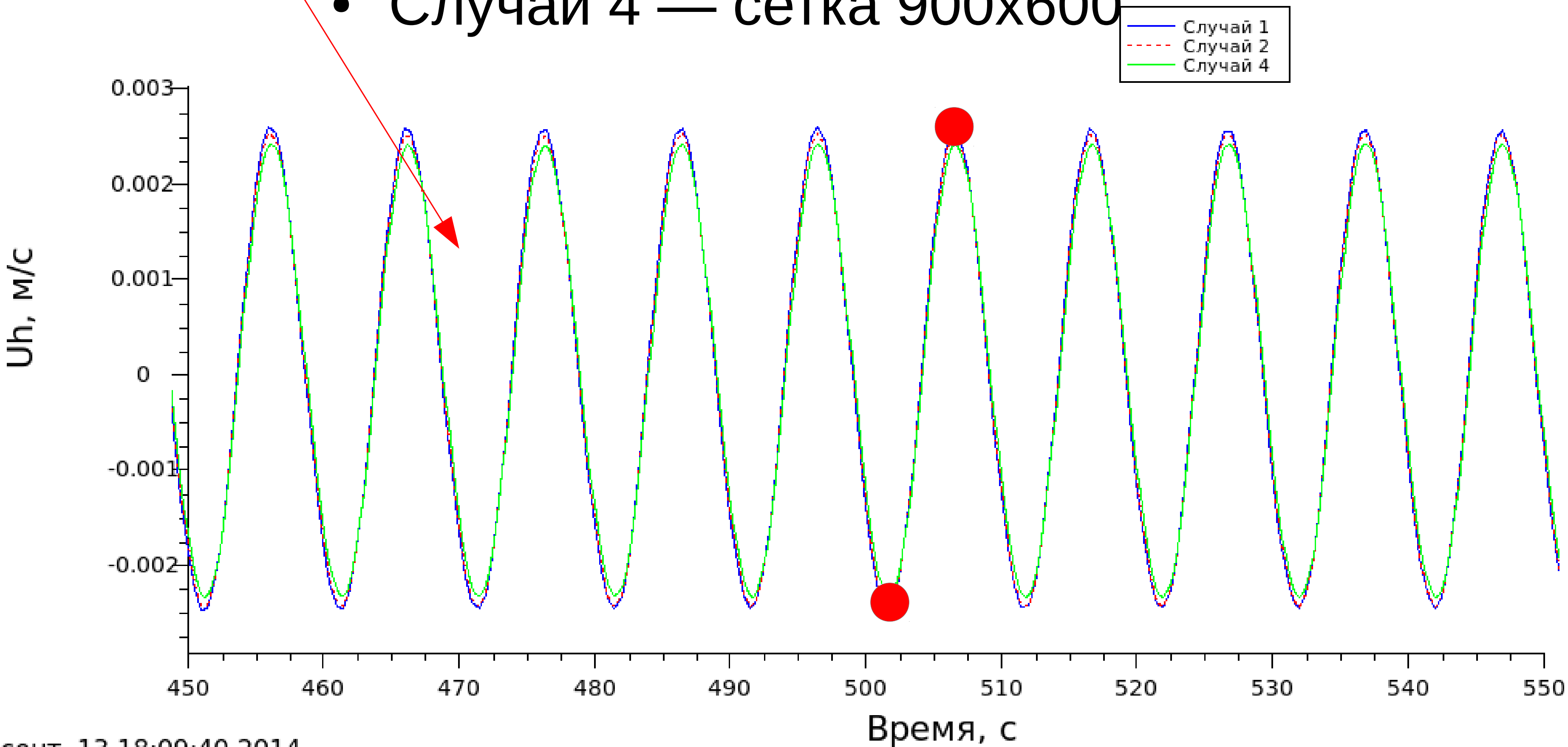
Результаты 1. Сеточная сходимость



Рабочая область

- Случай 1 — сетка 225x150
- Случай 2 — сетка 450x300
- Случай 4 — сетка 900x600

Красными кружками показаны моменты времени, для которых проводилось сравнение МКО и МСЭ



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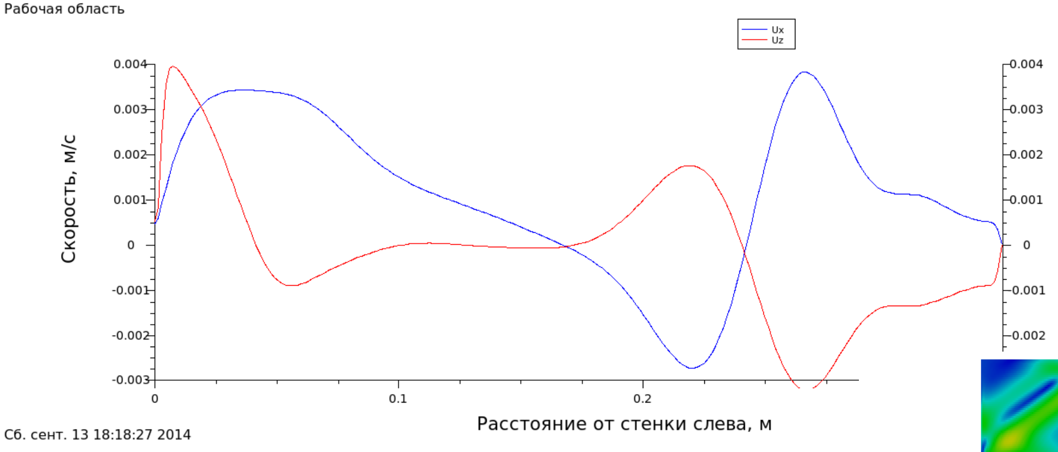
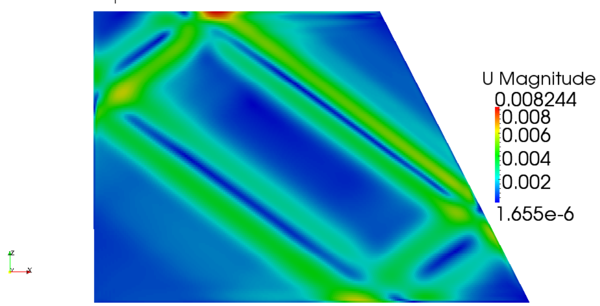
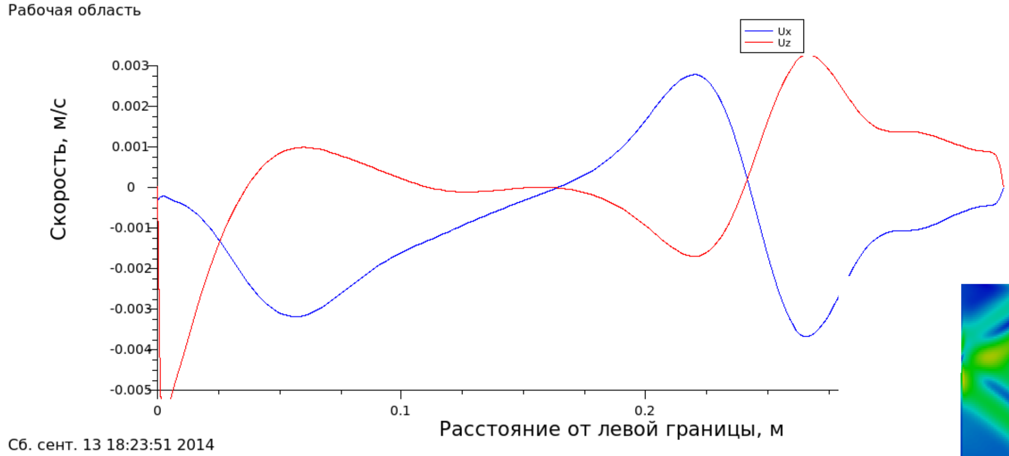
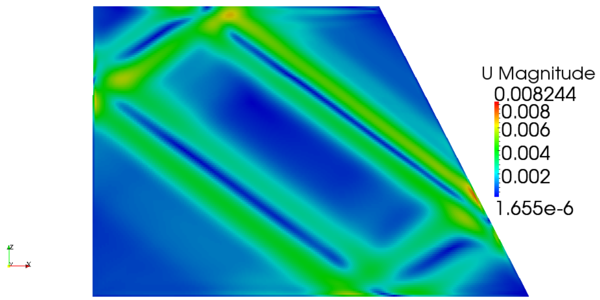
Результаты 2. Качественная картина





Профиль горизонтальной и вертикальной скоростей на линии y=20см для момента 501.5с (максимум горизонтальной скорости)

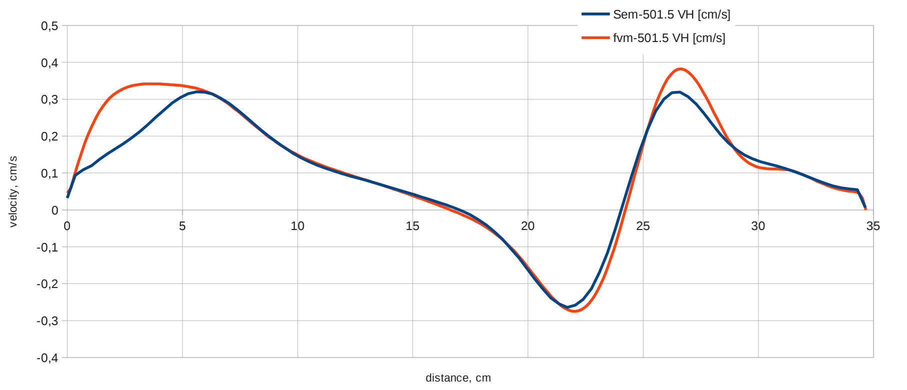


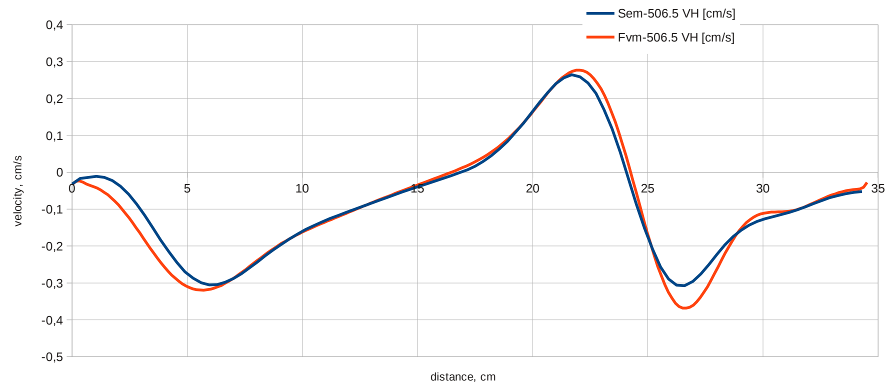


Профиль горизонтальной и вертикальной скоростей на линии y=20см для момента 506.5с (минимум горизонтальной скорости)

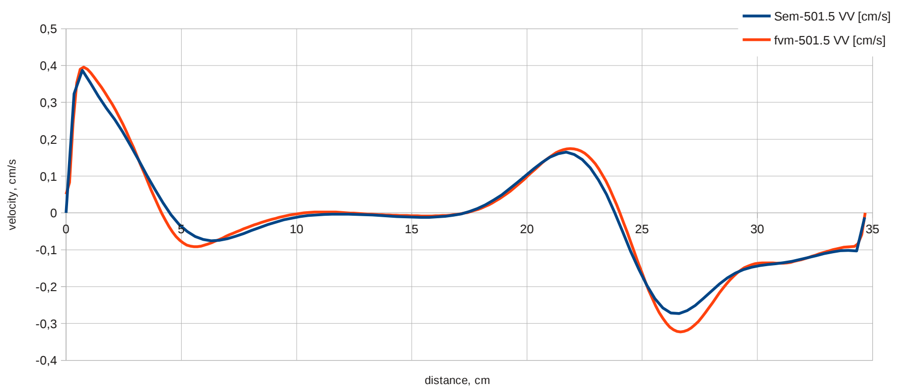
Результаты 3. МКО vs МСЭ

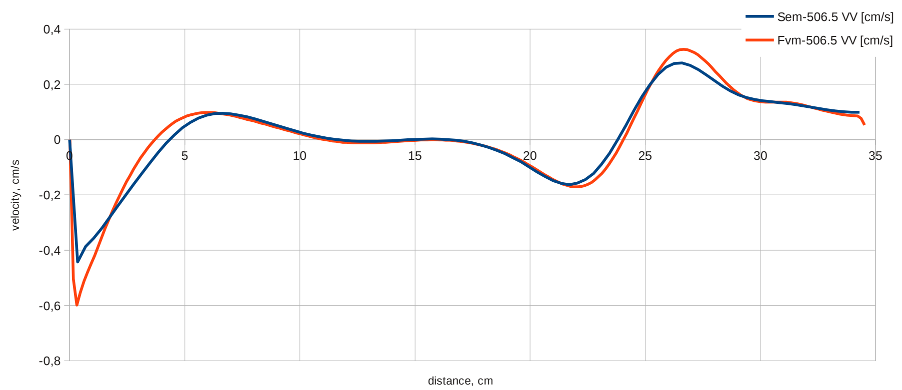
Красный — МКО, синий — МСЭ. Сравнивается распределение поля скорости вдоль линии y=20см





Горизонтальная скорость





Вертикальная скорость

t=501.5c

t=506.5c

Conclusion

1. With the help of 2D Direct Numerical Simulation the experiment on formation of the internal wave attractor was reproduced qualitatively.
2. Salinity profile profile changes were studied for long time intervals and its influence on shape of the attractor was demonstrated.
3. Qualitative similarity of the computations and experiments was possible only after taking into account the imperfect stratification near upper boundary.
4. Comparison of two different DNS approaches: FVM and SEM has been carried out.
5. Quantitative similarity of DNS and experiment requires 3D modelling

DNS was performed using open libraries NEK5000 (for SEM), OpenFOAM (for FVM) and other open source tools.

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