Paper 40

Analytical quasi-one-dimensional method and criteria for the transition from 2D/3D to 1D models for simulation of non-equilibrium two-phase turbulent flows dynamics

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INTRODUCTION

Current LPA codes are 1-D (e.g., two-fluid model (RELAP5) or drift flux model (RETRAN)).

Problem: how to take into account the real 2D (and 3-D) flow characteristics in 1-D models.

Typical solution: using flat profiles model -> field equations become 1-D. This provides high calculation speed and saves memory.

But, important distributed information is lost.

These lost parameters are important not only for simple tube geometry, but also annular and sub-channel geometry.

Dr. N. Zuber offered the classic solution to distribution parameter (DP) via C_0 for continuity equation, Drs. Hancox and Nicoll provided empirical extensions to energy and momentum equations.

This work presents *analytical derivation* of the DPs using power-mode approximation for the monotone (and non-monotone) profile of basic variables to the continuity, energy and momentum equations.

Conservation Laws and Averaging

<u>3-D Drift Flux Model</u>

$$\frac{\partial \rho}{\partial t} + \nabla\left(\rho \vec{u}\right) = 0; \qquad \rho\vec{u} \equiv \alpha_d \rho_d \vec{u}_d + \left(1-\alpha_d\right)\rho_c \vec{u}_c;$$

$$\frac{\partial \alpha_d \rho_d}{\partial t} + \nabla\left(\alpha_d \rho_d \vec{u}\right) = \Gamma_d - \nabla\left(\frac{\alpha_d \rho_d \rho_c}{\rho}\vec{u}_{dj}\right); \qquad \vec{u}_{dj} \equiv \vec{u}_d - \vec{j} = \left(1-\alpha_d\right)\left(\vec{u}_d - \vec{u}_c\right);$$

$$\frac{\partial \rho\vec{u}}{\partial t} + \nabla\left(\rho\vec{u}\vec{u}\right) = -\nabla P + \nabla\left(\bar{\bar{\tau}} + \bar{\bar{\tau}}^t - \frac{\alpha_d}{1-\alpha_d}\frac{\rho_d \rho_c}{\rho}\vec{u}_{dj}\vec{u}_{dj}\right) + \rho\vec{g}; \qquad \vec{j} \equiv \alpha_d \vec{u}_d + \left(1-\alpha_d\right)\vec{u}_c \equiv \vec{j}_d + \vec{j}_c;$$

$$\frac{\partial \rho h}{\partial t} + \nabla\left(\rho h \vec{u}\right) = -\nabla\left[\vec{q} + \vec{q}^t + \frac{\alpha_d \rho_d \rho_c}{\rho}\left(h_d - h_c\right)\vec{u}_{dj}\right] + \frac{\partial P}{\partial t} + \left[\vec{u} + \frac{\alpha_d\left(\rho_c - \rho_d\right)}{\rho}\vec{u}_{dj}\right]\nabla P + \Phi_{diss}$$

Analytical and simple model formulations
for $C_k \neq 1$, λ, St or h_q
(round pipe and slot channel)

Homogeneous model without fine parameter space structure:
$C_k \equiv 1$, λ, St or h_q

Analytical and simple model formulations
for $C_{kn} \neq 1$, λ_n, St_n or h_{qn}
(subchannel geometry)

<u>Quasi-1-D Drift Flux Model</u>

$$k = \begin{cases} f - fluid - 1 \\ g - gas - 2 \end{cases}$$

$$\frac{\partial m_g}{\partial t} + \frac{\partial(\rho_g \langle j_g \rangle)}{\partial z} = \langle \Gamma_i''' \rangle; \qquad \langle j_g \rangle \equiv \langle \alpha \rangle (\underline{C_0}\langle j \rangle + \bar{V}_{gj}), \text{ Zuber N.}; \qquad m_k \equiv \langle \alpha_k \rangle \rho_k;$$

$$\frac{\partial m_f}{\partial t} + \frac{\partial(\rho_f \langle j_f \rangle)}{\partial z} = -\langle \Gamma_i''' \rangle; \qquad \langle j_f \rangle = \langle j \rangle - \langle j_g \rangle; \qquad \langle \rho w \rangle \equiv \rho_f \langle j_f \rangle + \rho_g \langle j_g \rangle;$$

$$\frac{\partial}{\partial t}\langle \rho w \rangle + \frac{\partial}{\partial z}\left(\frac{\underline{C_{g3}}\rho_g \langle j_g \rangle^2}{\langle \alpha \rangle} + \frac{\underline{C_{f3}}\rho_f \langle j_f \rangle^2}{\langle 1-\alpha \rangle}\right) = -\frac{\partial P}{\partial z} - \langle F_w'''(\underline{\lambda},\ldots) \rangle - \Delta P_\delta' - g\langle \rho \rangle;$$

$$\frac{\partial}{\partial t}(\underline{C_{g1}} m_g \langle h_g \rangle + \underline{C_{f1}} m_f \langle h_f \rangle) + \frac{\partial}{\partial z}(\underline{C_{g2}}\rho_g \langle j_g \rangle \langle h_g \rangle + \underline{C_{f2}}\rho_f \langle j_f \rangle \langle h_f \rangle) = \langle q_w'''(\underline{h_q},\ldots) \rangle.$$

Figure 2a. Block-diagram of quasi-1-D model deriving, simple geometry.

PRESETATION GOALS (Part 1)

1. To construct a more complete and universal analytical formulations of closure relationships for the distribution parameters (DPs) C_{ks} ($k=f$ –fluid or g - vapor; s=0,1,2,3 - mass, energy, momentum) in non-equilibrium two-phase flows

$$C_{ks} = \frac{\langle \alpha_k \varphi_{ks} \rangle}{\langle \alpha_k \rangle \langle \varphi_{ks} \rangle} = \frac{A \int_A \alpha_k \varphi_{ks} dA}{\int_A \alpha_k dA \int_A \varphi_{ks} dA}$$

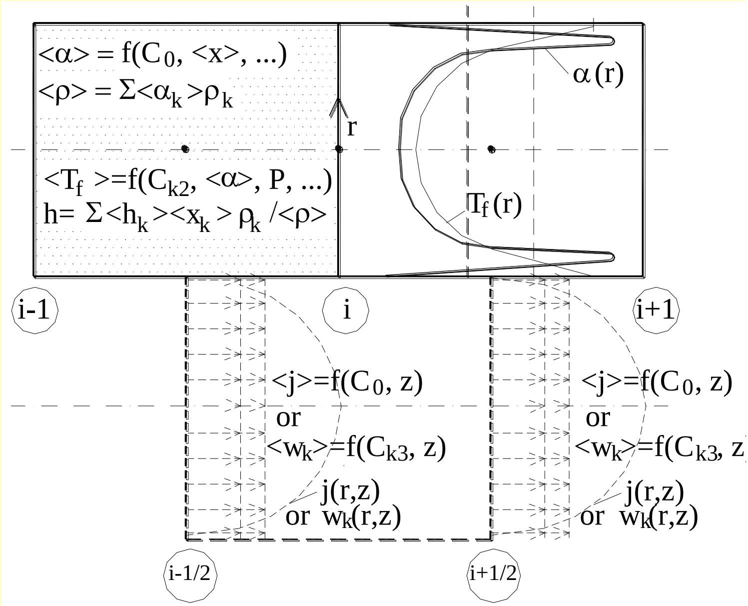
φ_{ks} = enthalpy h_k, or superficial j, or phase w_k (w_k^2) velocity.

2. To provide the representation of the integral formulations of these main effects that control the phase parameter distribution.

3. To introduce some examples of effects of radial variations of parameters on the above mentioned characteristics.

Control volume for mass
and energy (i) – (i+1)

$\langle\alpha\rangle$ $\langle T_f\rangle$ T_{fw}



Control volume for momentum
(i-1/2) - (i+1/2)

Fig. 1. Procedure of space discretization (for large control volume).

Definitions of distribution parameters C_{ks}

Drift flux model	Two-fluid model
$C_{k0} = \dfrac{\langle\alpha_k j\rangle}{\langle\alpha_k\rangle\langle j\rangle}$	$C_{k1} = \dfrac{\langle\alpha_k h_k\rangle}{\langle\alpha_k\rangle\langle h_k\rangle}$
$C_{k2}^{j} = \dfrac{\langle\alpha_k h_k j\rangle}{\langle\alpha_k\ j\rangle\langle h_k\rangle}$	$C_{k2} = \dfrac{\langle\alpha_k h_k w_k\rangle}{\langle\alpha_k w_k\rangle\langle h_k\rangle}$
$C_{k3}^{j} = \dfrac{\langle\alpha_k j^2\rangle}{\langle\alpha_k\rangle\langle j\rangle^2}$	$C_{k3} = \dfrac{\langle\alpha_k w_k^2\rangle}{\langle\alpha_k\rangle\langle w_k\rangle^2}$ *
$C_{k4}^{j} = \dfrac{\langle\alpha_k j^3\rangle}{\langle\alpha_k\rangle\langle j\rangle^3}$	$C_{k4} = \dfrac{\langle\alpha_k w_k^3\rangle}{\langle\alpha_k\rangle\langle w_k\rangle^3}$ **

* - Bussinesk coefficient, for $\alpha_k \equiv 1$

** - Koriolis coefficient, for $\alpha_k \equiv 1$

k=*f* –fluid or *k*=*g* - vapor

Main assumptions and properties of the derived quadrature relationships for DPs are:

(a) the use of the drift flux model,

(b) the quasi-steady-state approximation, and

(c) the power-mode approximations of the local distribution of the variables,

(d) two-zone accounting for heterogeneity of void fraction and enthalpy profiles in the channel cross-section.

1. These DPs C_{ks} quadrature are expressed in terms of elementary functions, they directly reflect the principle of superposition, generalize and unify not only the Zuber-Findlay method, but also Hancox-Nicoll and Hibiki-Ishii methods.

2. The revealed complementarity properties are particularly useful for the purposes of testing, validating and verifying DPs.

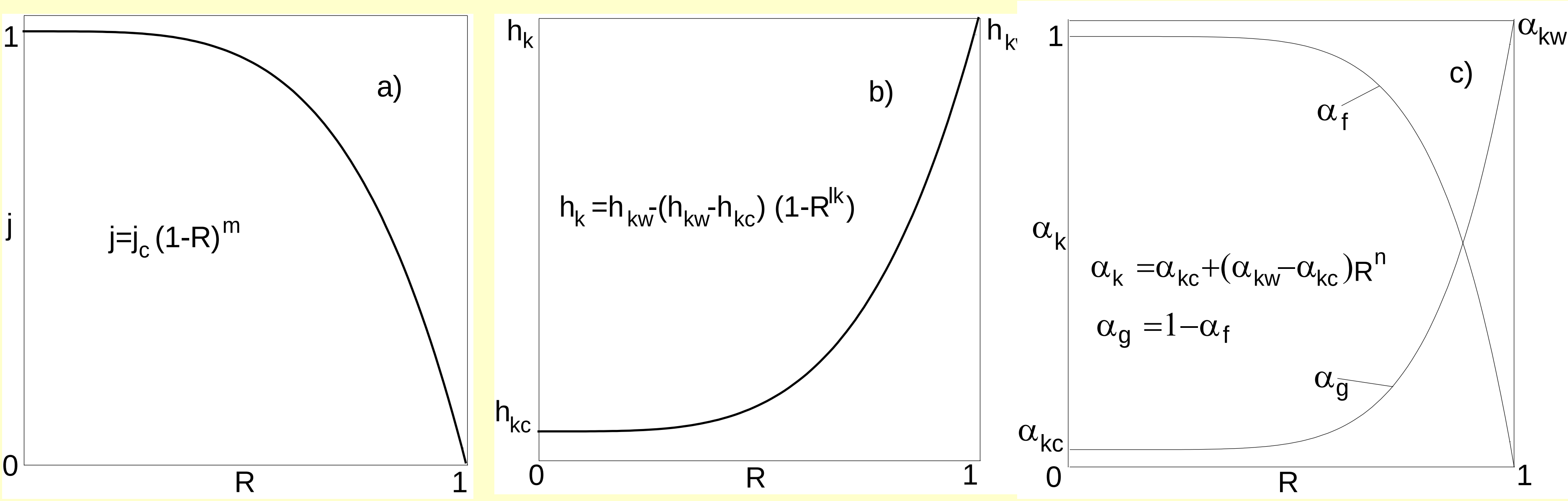


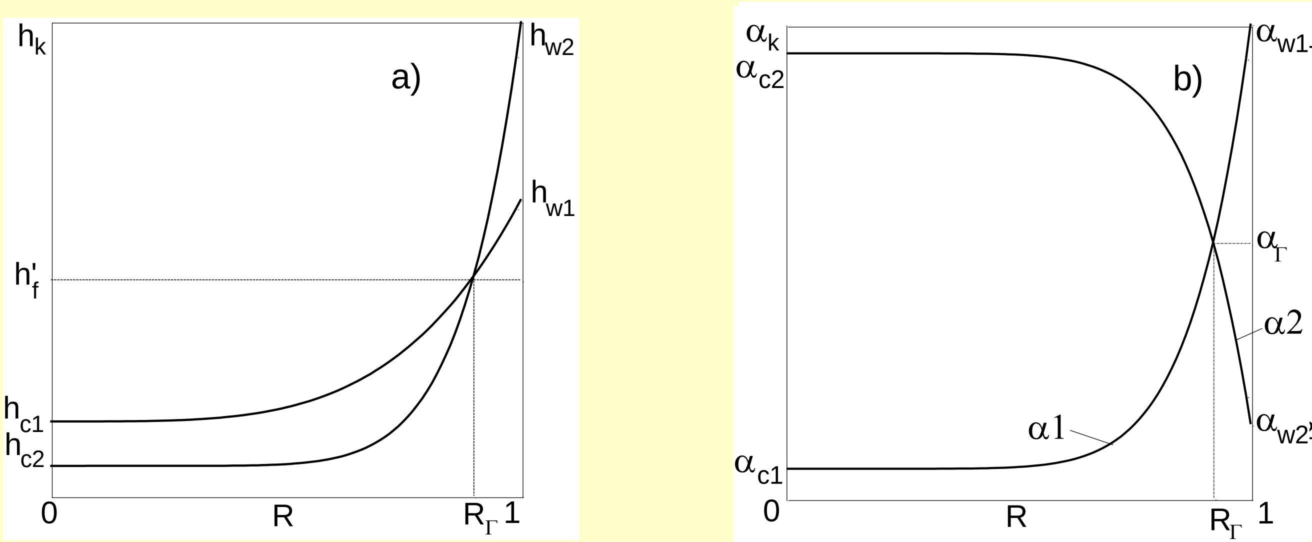
Figure 3. Power approximations for the parameter profiles: a) volumetric flux density; b) *k*-phase enthalpy - h_k, and c) *k*-phase void fraction - α_k.

A set of analytical relationships for C_{ki} were derived by inserting the power-mode approximation of the monotone variable profiles (Fig. 3) into the original definitions and integrating the linear combination of differential binomials.

Table 1. Functional forms of distribution parameters for 1-D conservation law equations of two-phase (two-component) flow for *monotonic* void profiles of k-phase ($k = g$; $k = f$).

…2-nd column DP … C_{k0} for k-phase is a part of each of the DP C_{ks}, controlling nature of the behavior of each. Obviously, the DP C_{k0} reduces to the classical DP Zuber-Findlay-C_0, when $k=g$ for void fraction.

DFM: $j = j_1 + j_2$; $j_k = \alpha_k w_k$; $j_{kj} = \alpha_k (w_k - j)$		COMPLIMENTARITY RELATIONS
Definition	Analytical relationship	
$C_{k0} = \dfrac{\langle \alpha_k j \rangle}{\langle \alpha_k \rangle \langle j \rangle}$ $C_{kh} = \dfrac{\langle h_k j \rangle}{\langle h_k \rangle \langle j \rangle}$	$C_{k0} = 1 + \dfrac{\gamma}{m+n+\gamma}\left(1 - \dfrac{\alpha_{kw}}{\langle \alpha_k \rangle}\right)$ $C_{kh} = 1 + \dfrac{\gamma}{l_k + m + \gamma}\left(1 - \dfrac{h_{kw}}{\langle h_k \rangle}\right)$	Balances between phases $1 \equiv \sum_{k=1}^{2} C_{k0} \langle \alpha_k \rangle$, where $\langle \alpha \rangle = \int_0^1 \alpha \gamma R^{\gamma-1} dR$
$C_{k1} = \dfrac{\langle \alpha_k h_k \rangle}{\langle \alpha_k \rangle \langle h_k \rangle}$	$C_{k1} = 1 + \dfrac{\gamma\left(1 - \dfrac{\alpha_{kw}}{\langle \alpha_k \rangle}\right)}{\ell_k + n + \gamma}\left(1 - \dfrac{h_{kw}}{\langle h_k \rangle}\right)$	$\sum_{k=1}^{2} C_{k1} \langle c_k \rangle \langle h_k \rangle = h_{e1}$ $h_{e1} = \langle \sum_{k=1}^{2} c_k h_k \rangle$
$C_{k2}^{j} = \dfrac{\langle \alpha_k h_k j \rangle}{\langle \alpha_k j \rangle \langle h_k \rangle}$ $C_{k2} = \dfrac{\langle \alpha_k h_k w_k \rangle}{\langle \alpha_k w_k \rangle \langle h_k \rangle}$	$C_{k2}^{j} = 1 + \left(C_{kh} - 1\right)\left(1 + F_k \dfrac{C_{k0} - 1}{C_{k0}}\right)$ $C_{k2} = \left(C_{k2}^{j} + \dfrac{C_{k1}}{C_{k0}} \widetilde{W}_{kj}\right) \dfrac{1}{1 + \widetilde{W}_{kj}}$	$\sum_{k=1}^{2} C_{k2} \langle X_k \rangle \langle h_k \rangle = h_{e2}$ $h_{e2} = \int_0^z q_w'''(z) dz \Big/ \langle \rho w \rangle$
$C_{k3}^{j} = \dfrac{\langle \alpha_k j^2 \rangle}{\langle \alpha_k \rangle \langle j \rangle^2}$ $C_{k3} = \dfrac{\langle \alpha_k w_k^2 \rangle}{\langle \alpha_k \rangle \langle w_k \rangle^2}$	$C_{k3}^{j} = 1 + \dfrac{2m + n + 2\gamma}{2\gamma m}\left(C_{k0} - 1\right)$ $C_{k3} = \dfrac{C_{k3}^{j} + 2C_{k0}\widetilde{W}_{kj} + \widetilde{W}_{kj}^2}{(1 + \widetilde{W}_{kj})^2}$	$1 \equiv \sum_{k=1}^{2} C_{k3}^{j} \langle \alpha_k \rangle$ $0 \equiv \sum_{k=1}^{2} \widetilde{W}_{kj} \langle \alpha_k \rangle$
$\widetilde{W}_{kj} = \dfrac{\langle \alpha_k w_{kj} \rangle}{\langle \alpha_k \rangle \langle j \rangle}$ $\langle X_k \rangle = \dfrac{\langle \alpha_k \rho_k w_k \rangle}{\langle \rho w \rangle}$	$c_k = \alpha_k \rho_k / \rho$; $\langle \widetilde{j} \rangle = \langle j \rangle / \langle \rho w \rangle$ $\rho = \sum_{k=1}^{2} \alpha_k \rho_k$; $\rho w = \sum_{k=1}^{2} \alpha_k \rho_k w_k$	$1 \equiv \langle \widetilde{j} \rangle \sum_{k=1}^{2} \rho_k \langle \alpha_k \rangle C_{k0}^{w}$ $C_{k0}^{w} = C_{k0} + \langle \widetilde{W}_{kj} \rangle$
$\gamma = \begin{cases} 1 - \mathit{flat\ channel} \\ 2 - \mathit{round\ tube} \end{cases}$	$F_k = \dfrac{(m+\gamma)^2}{l_k n}\left[\dfrac{(l_k+\gamma)(n+\gamma)(l_k+m+\gamma)(m+n+\gamma)}{\gamma(m+\gamma)(l_k+n+\gamma)(l_k+m+n+\gamma)} - 1\right]$	



a) for enthalpies (*Bh*)

b) for phase void fractions (*B*)

$$h = \begin{cases} h_f = h_{c1} + (h_{w1} - h_{c1})R^{l_1}, \ 0 \le R \le R_\Gamma \\ h_g = h_{c2} + (h_{w2} - h_{c2})R^{l_2}, R_\Gamma \le R \le 1 \end{cases}$$

$$\alpha = \begin{cases} \alpha_{c1} + (\alpha_{w1} - \alpha_{c1})R^{n_1}, \quad 0 \le R \le R_\Gamma \\ \alpha_{c2} + (\alpha_{w2} - \alpha_{c2})R^{n_2}, \quad R_\Gamma \le R \le 1 \end{cases}$$

Fig. 4. Model of two monotonic profile superposition ("B").

An analogous set of analytical relationships was derived for the case of non-monotonous variable profile (Fig. 4), including a compound channel or sub-channel (see Table 2) in the form of so-called two-zone representation of two monotone "cross-linked" on the border line R_Γ of two power-mode approximations.

Table 2. Functional forms of distribution parameters for one-dimensional conservation law equations of two-phase flow for the case of *non-monotonic* profiles.

Definition	Analytical relationship	COMPLIMENTARITY RELATION
$C_{k0}^{B}=\dfrac{\langle\alpha_k^B j\rangle}{\langle\alpha_k^B\rangle\langle j\rangle}$ $C_{kh}^{Bh}=\dfrac{\langle h_k^B j\rangle}{\langle h_k^B\rangle\langle j\rangle}$	$C_{k0}^{B}=1+\dfrac{\gamma}{m}\left(1-F_m^{\alpha}\dfrac{m+\gamma}{\gamma}\right)$ $C_{kh}^{Bh}=1+\dfrac{\gamma}{m}\left(1-F_m^{h}\dfrac{m+\gamma}{\gamma}\right)$	Balances between phases $1\equiv\sum_{k=1}^{2}C_{k0}\langle\alpha_k\rangle$
$C_{k1}^{B}=\dfrac{\langle\alpha_k^B h_k\rangle}{\langle\alpha_k^B\rangle\langle h_k\rangle}$ $C_{k1}^{Bh}=\dfrac{\langle\alpha_k h_k^B\rangle}{\langle\alpha_k\rangle\langle h_k^B\rangle}$	$C_{k1}^{B\alpha}=1+\dfrac{\gamma}{l_k}\left(1-F_{l_k}^{\alpha}\dfrac{l_k+\gamma}{\gamma}\right)\left(1-\dfrac{h_{kw}}{\langle h_k\rangle}\right)$ $C_{k1}^{Bh}=1+\dfrac{\gamma}{n_k}\left(1-F_{n_k}^{h}\dfrac{n_k+\gamma}{\gamma}\right)\left(1-\dfrac{\alpha_{kw}}{\langle\alpha_k\rangle}\right)$	
$C_{k2}^{jB}=\dfrac{\langle\alpha_k^B h_k j\rangle}{\langle\alpha_k^B j\rangle\langle h_k\rangle}$ $C_{k2}^{jBh}=\dfrac{\langle\alpha_k h_k^B j\rangle}{\langle\alpha_k j\rangle\langle h_k^B\rangle}$	$C_{k2}^{jB}=1+(C_{kh}-1)\dfrac{m+\gamma}{l_k}\left[1-\left(F_{l_k}^{\alpha}-F_{l_k+m}^{\alpha}\right)\dfrac{(l_k+m+\gamma)(l_k+\gamma)}{\gamma\, mC_{k0}^{B}}\right]$ $C_{k2}^{jBh}=1+\left(C_{k0}^{B}-1\right)\dfrac{m+\gamma}{n_k}\left[1-\left(F_{n_k}^{h}-F_{n_k+m}^{h}\right)\dfrac{(n_k+m+\gamma)(n_k+\gamma)}{\gamma\, mC_{kh}^{Bh}}\right]$	
$C_{k3}^{jB}=\dfrac{\langle\alpha_k^B j^2\rangle}{\langle\alpha_k^B\rangle\langle j\rangle^2}$ $C_{k3}^{B}=\dfrac{\langle\alpha_k^B w_k^2\rangle}{\langle\alpha_k^B\rangle\langle w_k\rangle^2}$	$C_{k3}^{jB}=\left(\dfrac{m+\gamma}{m}\right)^2\left(1-2F_m^{B\alpha}+F_{2m}^{B\alpha}\right)$ $C_{k3}^{B}=\dfrac{C_{k3}^{jB}+2C_{k0}^{B}\widetilde{W}_{kj}+\widetilde{W}_{kj}^{2}}{(1+\widetilde{W}_{kj})^2}$	$1\equiv\sum_{k=1}^{2}C_{k3}^{j}\langle\alpha_k\rangle$
$\widetilde{W}_{kj}=\dfrac{\langle\alpha_k^B w_{kj}\rangle}{\langle\alpha_k^B\rangle\langle j\rangle}$	$\widetilde{W}_{kj}=\dfrac{b[1-\langle\alpha_g^B\rangle(1-\rho^*)C_{g0}^{B}]}{1-D+b\langle\alpha_g^{B\alpha}\rangle(1-\rho^*)};\ b=\dfrac{\overline{W}_{kj}\rho_f}{G_A};\ \rho^*=\dfrac{\rho_g}{\rho_f}$	

$$F_{\xi k}^{\alpha}=\frac{\gamma}{\langle\alpha_k^B\rangle}\left[\frac{\alpha_{kc2}}{\xi_k+\gamma}+\frac{\alpha_{kw2}-\alpha_{kc2}}{\xi_k+n_2+\gamma}+\left(\frac{\alpha_{kc1}-\alpha_{kc2}}{\xi_k+\gamma}+\frac{\alpha_{k\Gamma}-\alpha_{kc1}}{\xi_k+n_1+\gamma}+\frac{\alpha_{k\Gamma}-\alpha_{kc2}}{\xi_k+n_2+\gamma}\right)R_{\Gamma}^{\xi_k^{\alpha}+\gamma}\right]$$

$$F_{\xi k}^{h}=\frac{\gamma}{\langle h_k^B\rangle}\left[\frac{h_{kc2}}{\xi_k+\gamma}+\frac{h_{kw2}-h_{kc2}}{\xi_k+l_2+\gamma}+\left(\frac{h_{kc1}-h_{kc2}}{\xi_k+\gamma}+\frac{h_{k\Gamma}-h_{kc1}}{\xi_k+l_1+\gamma}+\frac{h_{k\Gamma}-h_{kc2}}{\xi_k+l_2+\gamma}\right)R_{\Gamma}^{\xi_k^{h}+\gamma}\right],$$

where $\xi_k^{\alpha}=\begin{cases} m,\ for\,C_{k0}^{B};\ l_k+m,\ for\,C_{k2}^{B\alpha} \\ l_k,\ for\,C_{k1}^{B\alpha};\ 2m,\ for\ C_{k3}^{B\alpha}.\end{cases}$; $\xi_k^{h}=\begin{cases} m,\ for\,C_{kh}^{Bh};\ n_k+m,\ for\,C_{k2}^{Bh} \\ n_k,\ for\,C_{k1}^{Bh};\ 2m,\ for\ C_{k3}^{B}.\end{cases}$

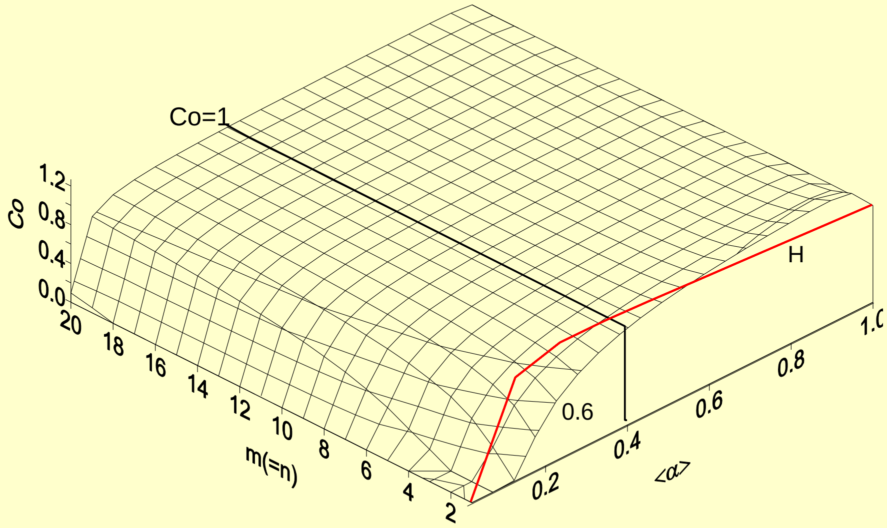


Figure 5. Distribution parameter C_0. for convex and concave void fraction profiles; α_w=0.4; H – Hancox & Nicoll [5].

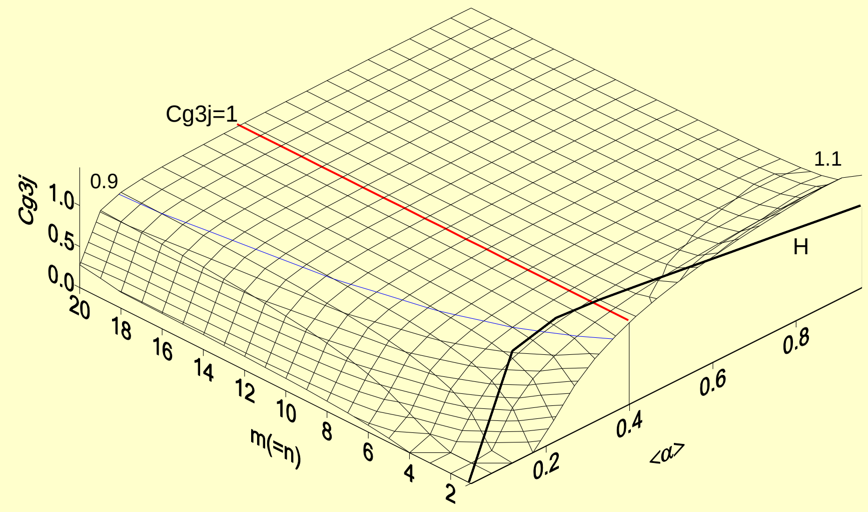


Figure 6. Distribution parameter for momentum flux $C_{g3}^{\ j}$, α_w=0.4; H – Hancox & Nicoll [5].

The two-phase flow parameter C_0 is included in the structure of each distribution parameter affecting the character of their behavior. Consequently, the main characteristics of the C_0 may serve as the basis for investigating the behavior of other distribution parameters as well in the general case of non-monotonic void fraction profiles. The comparison with Hancox-Nicoll empirical relationship is shown in the Figs. 5 and 6. Though the comparison rather satisfactory, but one can see the vast expanses of differences from each other and from unity.

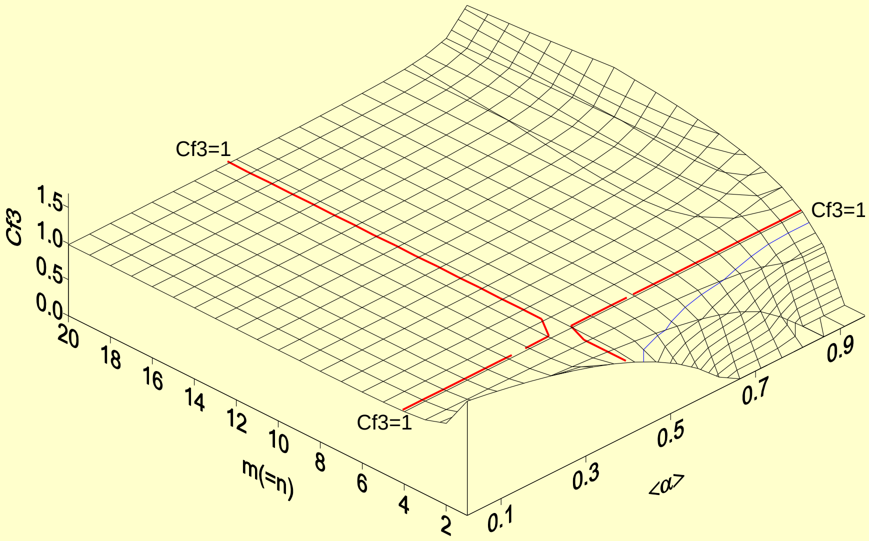


Figure 7. Distribution parameter for momentum flux C_{f3}, for α_w=0.4, $\tilde{W}_{fj}$ = 0.5.

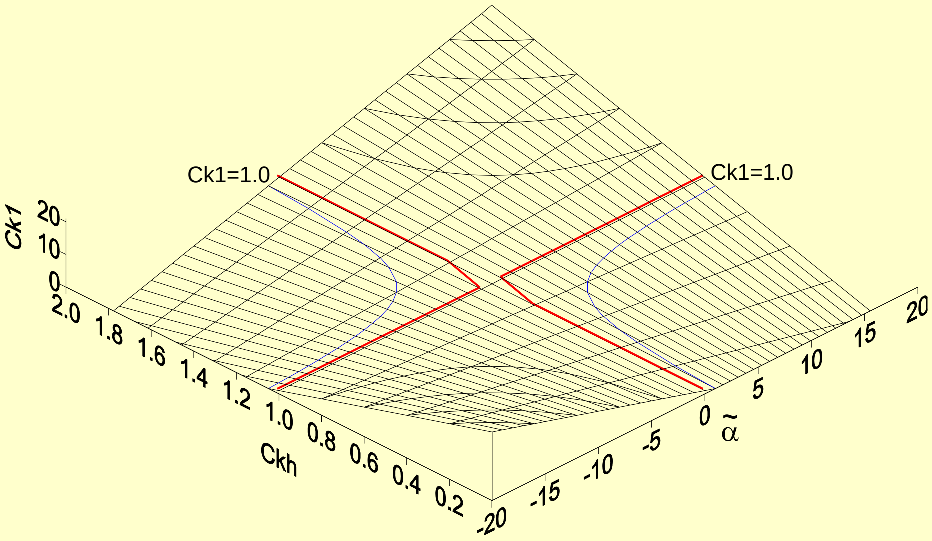


Figure 8. Distribution parameter for enthalpy C_{k1} as function of C_{kh} and form factor $\tilde{\alpha} = 1 - \alpha_{kw}/\langle\alpha_k\rangle$.

Due to the hierarchical structure of the obtained analytical relationships for distribution parameters it is possible to build the more complicated distribution parameters (for example, energy and momentum equation components) as the function of more simple distribution parameters (for example, C_{k0} and C_{kh}).

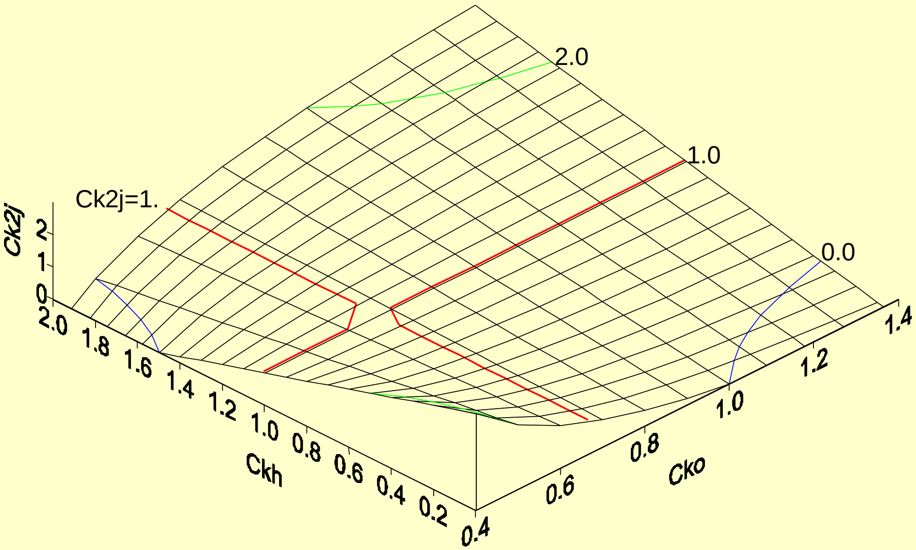


Fig. 9. Distribution parameter for enthalpy flux $C_{k2}^{\,j}$ as function of C_{kh} and C_{k0}, for F_k=2.

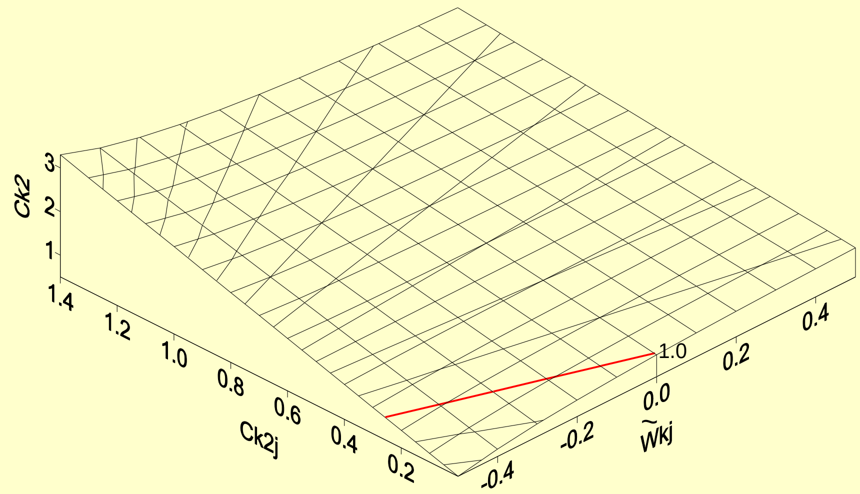


Fig. 10. Distribution parameter for enthalpy flux C_{k2} as function of $C_{k2\mathrm{j}}$ for $C_{k1}/C_{k0} = 0.5$.

The property of the hierarchical structure provides the most laconic and universal presentations of the compound parameter distributions, see Figs. 7-10. These Figures illustrate the vast expanses of differences from the unity for distribution parameters. This fact points out the *invalidity to use flat profile approximation* in the 1-D model for number of the non-equilibrium flow regimes, in particularly, for “subcooled” flow boiling and for the “post critical” heat transfer.

$$C_{k0}^{B} = 1+\frac{\gamma}{m+n+\gamma}\left\{\left[1+\frac{n+\gamma}{m}\left(1-R_{\Gamma}^{m}\right)\right]\left(1-\frac{\alpha_{kw2}}{\langle\alpha_k^B\rangle}\right)+\ \frac{n}{m}\cdot\frac{1-R_{\Gamma}^{m}}{1-R_{\Gamma}^{n}}\cdot\frac{\alpha_{kw2}-\alpha_{k\Gamma}}{\langle\alpha_k^B\rangle}\right\}$$

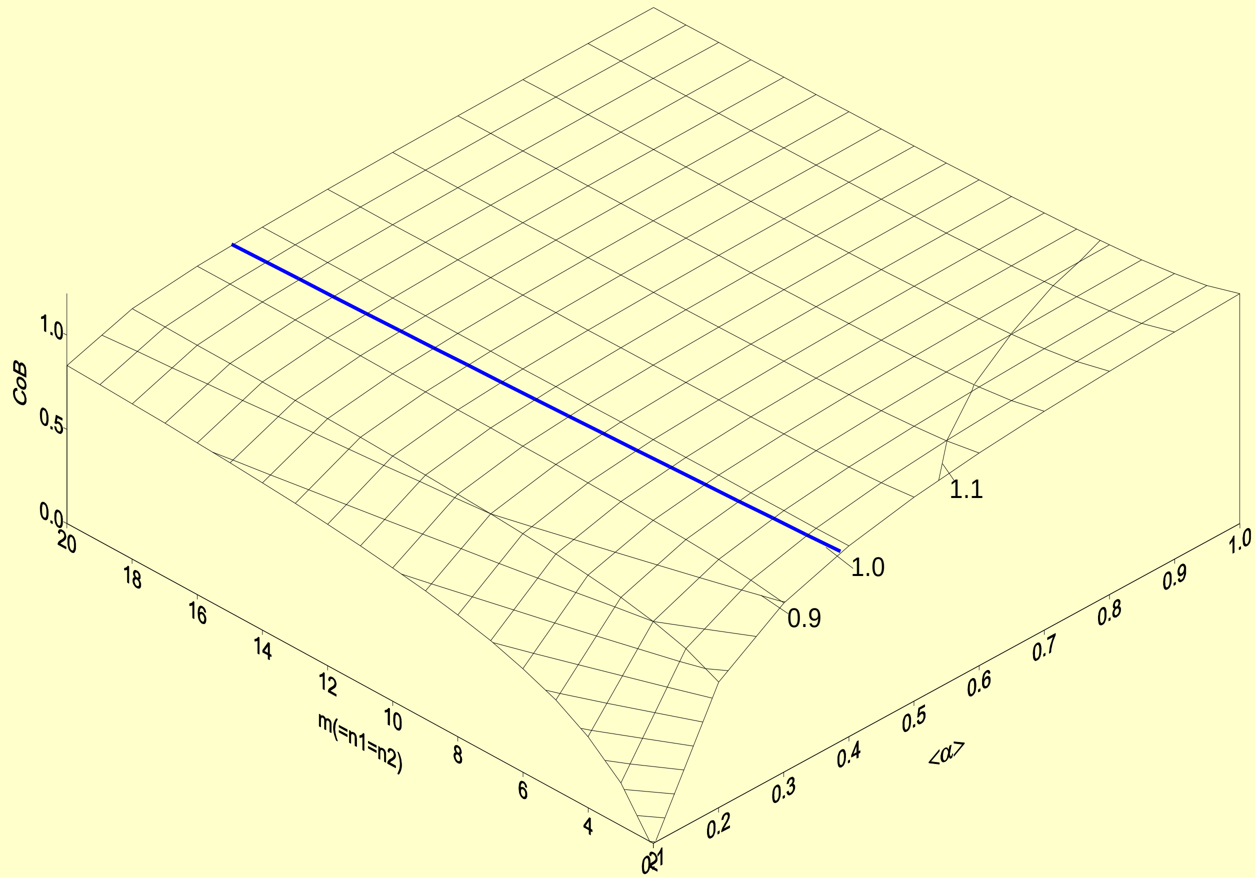
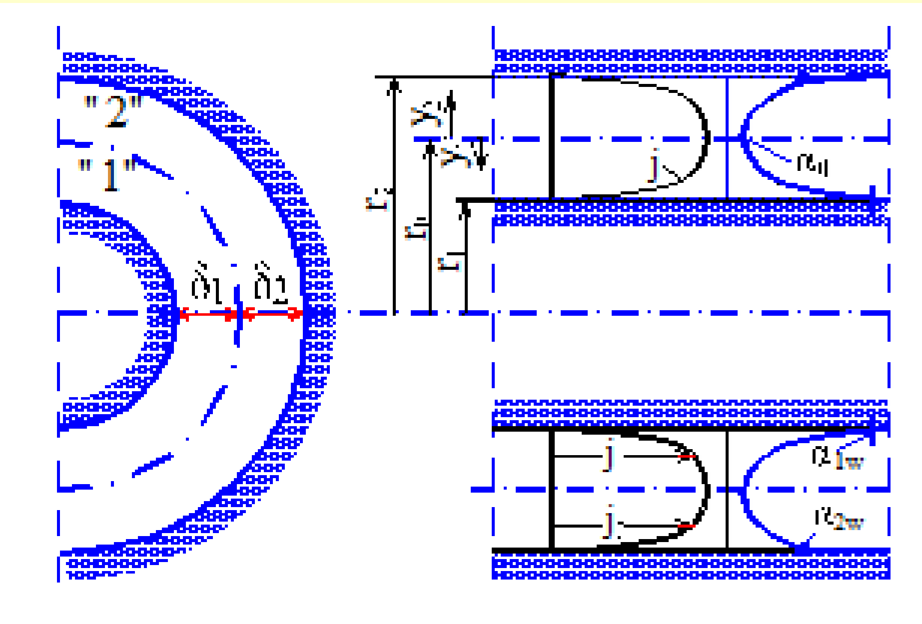
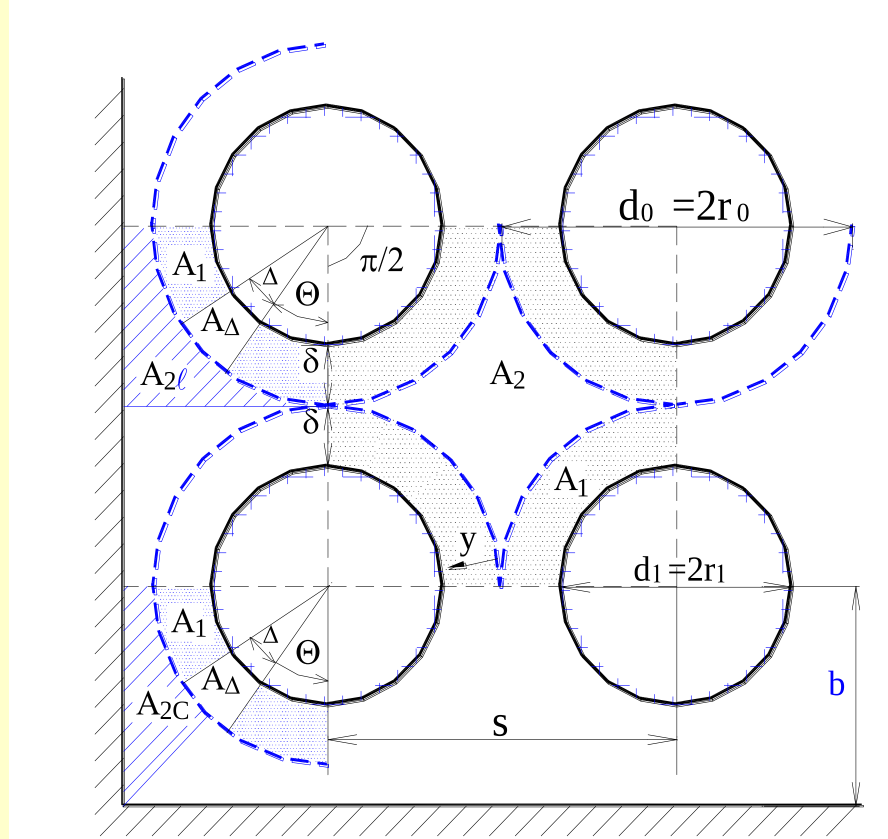


Fig. 12. Distribution parameter $C_{k0}{}^{B}$ for non-monotonic void profiles of two-phase flow when R_{Γ}=0.98; α_{w2}=0.2; α_{Γ}=0.4

Annular and pin bundle geometries





$$C_{kaa} = C_{kaab}\frac{\left(1 \mp \tilde{\delta}_a\left(1 \mp \tilde{\delta}_a \dfrac{C_{kaap}}{C_{kaab}}\dfrac{\langle \alpha_k \rangle_{ap}}{\langle \alpha_k \rangle_{ab}}\dfrac{\langle \varphi_{ka} \rangle_{ap}}{\langle \varphi_{ka} \rangle_{ab}}\right)\right.}{\left(1 \mp \tilde{\delta}_a \dfrac{\langle \alpha_k \rangle_{ap}}{\langle \alpha_k \rangle_{ab}}\right)\left(1 \mp \tilde{\delta}_a \dfrac{\langle \varphi_{ka} \rangle_{ap}}{\langle \varphi_{ka} \rangle_{ab}}\right)}$$

$$C^{\Lambda}_{g01} = \frac{\Delta \cdot (1-\tilde{\delta}) \cdot \int_{\theta}^{\theta+\Delta}\int_0^1 \alpha_1 j_1 (1-2\tilde{\delta}Y_1)\,dY\,d\theta}{\int_{\theta}^{\theta+\Delta}\int_0^1 \alpha_1 (1-2\tilde{\delta}Y_1)\,dY\,d\theta \int_{\theta}^{\theta+\Delta}\int_0^1 j_1 (1-2\tilde{\delta}Y_1)\,dY\,d\theta}$$

CONCLUSIONS (Part 1)

A set of analytical relationships for DPs C_{ks} was derived with power-mode approximation of the monotone variable profiles and integrating of the linear combination of differential binomials, which were expressed in terms of elementary functions. There are generalize and unify not only the Zuber-Findlay, but also the Hancox-Nicoll and Hibiki-Ishii methods. An analogous set of relationships was derived for non-monotonous profiles, including a compound channel or sub-channel.

These integral forms of the DPs make up the interrelation of the hierarchical structure between continuity, energy and momentum conservation law equations. Moreover, kinematic (i.e., simple form) DPs, such as C_{k0} and C_{k1}, are a part of more compound DPs for an energy transfer C_{k2} and momentum transfer C_{k3} relationships and affect in many respects the character of their behavior.

The system of the DPs reciprocal products and the *k*-phase average contents for the quasi-1-D model are derived. These complementation properties reflect the integral balances mass, enthalpy, momentum and their fluxes. In turn, it is a consequence of the unified consideration of DPs for each phase through its volume fraction: as α- void, or $(1-\alpha)$- fluid fraction. These integral balances between phases are useful both to the quasi-1-D theories of two-phase flow modeling, and to semi-empirical applications, including testing and verification problems for the C_{ks} closure relationships.

Paper 40 part 2

LYON-TYPE INTEGRAL FORMS OF WALL FRICTION, HEAT- AND MASS TRANSFER CLOSURE RELATIONSHIPS FOR NON-EQUILIBRIUM TWO-PHASE FLOWS. GENERALIZATION FOR ANNULAR AND ROD CLUSTER GEOMETRIES

Main assumptions of the derived closure relationships for friction, heat- and mass-transfer factors:

(a) coolant flows (with high aspect ratio of length to diameter), which occur in the frames of the boundary layer various models,

(b) the quasi-steady-state approximation,

(c) the use of the drift flux model,

(d) the phenomenological theory of hydrodynamics, heat, and mass transfer - gradient hypotheses (Fick's, Fourier's and Newton's) are used to describe the substance, heat, and momentum fluxes

(e) the generalized of variables separation method (A.D. Polyanin)

Field equations of:	Definitions:
Mixture mass $$\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\vec{u})=0,$$	$\rho=(\alpha\rho)_g+(\alpha\rho)_f$; $\rho\vec{u}=(\alpha\rho\vec{u})_g+(\alpha\rho\vec{u})_f$
Convective diffusion, $\vec{N}_T=\vec{N}_t+\vec{N}_d$ $$\rho\frac{\partial c}{\partial t}+\rho\vec{u}\cdot\nabla c=-\nabla\cdot(\vec{N}_T)+\Gamma$$	$c=(\alpha\rho)_g/\rho$; $\vec{N}_d=c(\alpha\rho)_g\vec{u}_{gf}$
Mixture energy $\vec{q}_T=\vec{q}_t+\vec{q}_d$ $$\rho\frac{\partial h}{\partial t}+\rho\vec{u}\cdot\nabla h=-\nabla\cdot(\vec{q}_T)+q_v$$	$h=[(\alpha\rho h)_g+(\alpha\rho h)_f]/\rho$ $\vec{q}_d=c\rho_f\vec{u}_{gf}(h_g-h_f)$
Mixture motion $\bar{\bar{\tau}}_T=\bar{\bar{\tau}}_t+\bar{\bar{\tau}}_d$ $$\rho\frac{\partial\vec{u}}{\partial t}+\rho\vec{u}\cdot\nabla\vec{u}=-\nabla\cdot(\tilde{\tilde{\tau}}_T+P)+\rho\vec{g}$$	$\vec{u}=[(\alpha\rho\vec{u})_g+(\alpha\rho\vec{u})_f]/\rho$ $\bar{\bar{\tau}}_d=\frac{c}{1-\alpha}\rho_f\vec{u}_{gf}\vec{u}_{gf}$
General form $$\rho\frac{\partial\varphi}{\partial t}+\rho\vec{u}\cdot\nabla\varphi=-\nabla\cdot(\tilde{\tilde{J}})+I_v$$	variable $\varphi\to(c, h, w)$; flux $\bar{\bar{J}}\to(\vec{N},\ \vec{q},\ \bar{\bar{\tau}})$; source $I_v\to(\Gamma,q_v,\rho\vec{g})$

Tab. 2: Non-conservative (transportable) forms of conservation law equations

The mathematical descriptions of similarity among the three transfer processes mentioned above (see the top line in Tab. 1) make it possible to introduce a formally generalized equation, in which the substance flux J is expressed by means of the total transfer characteristic ε_T and the gradient of the transfer potential φ normal to the wall as follows:

$$J = \rho \varepsilon_T \, \partial\varphi / \partial y \,, \qquad (1)$$

where $\varepsilon_T = \varepsilon + \varepsilon_t$ - is the total (molecular (ε) + turbulent (ε_t)) substance transfer characteristic (coefficient), namely, viscosity, thermal diffusivity or diffusion coefficient (see the 2 -nd line of Tab. 1).

After scaling variables in equation (1) with respect to their wall values and after integrating along radius Y, we can obtain the profile of variation for any of the potentials under consideration in the channel cross-section, if the substance flux and molecular + turbulent transfer characteristics are known. This is expressed by the formula:

$$\varphi_w^+ - \varphi^+ = Pe_{\varphi *} \int_0^Y \frac{\widetilde{J}}{\left(\widetilde{\rho}\widetilde{\varepsilon}\right)_T} dY' \qquad (2)$$

The detailed description of the substance fluxes and the key to decode the designations are obvious from the first six lines of the Tab.1. Using the definitions given in Tab. 1, one can easily reconstruct specific relationships for the profiles of axial velocity, enthalpy (temperature), and concentrations from the integral (2), see the 3-d line.

Separate effects (e=1 ÷6- is used to identify the components , e=1÷4 pipe/flat channel) are explained in the Tab.3:

$$\frac{1}{r}\frac{\partial}{\partial r}(rJ_{r\theta}) = \underset{e=1\downarrow}{I_{v\theta}} + \underset{e=2\downarrow}{\rho w\frac{\partial\varphi}{\partial z}} + \underset{e=3\downarrow}{\rho v\frac{\partial\varphi}{\partial r}} - \underset{e=4\downarrow}{\rho\frac{\partial\varphi}{\partial t}} + \underset{e=5\downarrow}{\frac{\rho v_\theta}{r}\frac{\partial w}{\partial\theta}} - \underset{e=6\downarrow}{\frac{\partial J_\theta}{r\partial\theta}}, \quad (10)$$

where $r=r_1+y$, w , v and v_θ are axial radial and azimuthal velocities; and J_θ is the azimuthal substance flux.

After variables scaling in the equation (10) for local substance flux, integrating it over cross section of pipe and each of zone annular and sub-channel, first with the variable upper limit Y_n, and then up to the wall Y_n=1, and, joining the obtained integrals (using Tab.3 designations), we have:

for pipe

$$\tilde{J}_{\sigma} = \frac{\Re_{\sigma}}{R^{\gamma-1}}\left(1-\sum_{e}\Phi_{e\varphi}K_{e\varphi}\right), \tag{11}$$

for annular

$$\tilde{J}_{a} = \frac{1\mp 2\tilde{\delta}_{\varphi a}}{1\mp 2\tilde{\delta}_{\varphi a}Y_{a}}\Re_{a}\left(1-\frac{\mp 2\tilde{\delta}_{\varphi a}}{1\mp 2\tilde{\delta}_{\varphi a}}\sum_{e}\Phi_{e\varphi a}K_{e\varphi a}\right), \tag{12}$$

and for sub-channel

$$\tilde{J}_{n}(Y,\theta) = \frac{1-2\tilde{\delta}_{\varphi n}}{1-2\tilde{\delta}_{\varphi n}Y_{n}}\hat{\Re}_{n}\left[\tilde{J}_{wn}(\theta)+\frac{2\tilde{\delta}_{\varphi n}}{1-2\tilde{\delta}_{\varphi n}}\sum_{e}\Phi_{e\varphi n}K_{e\varphi n}\right]. \tag{13}$$

Averaged transfer components - $\Phi_{e\varphi}$, which are generalized mass forces in the form of component Froude numbers are given in the 4th column of Tab. 3.

	2 Friction factor, heat and mass transfer coefficients	**3** Form-factors of source/sink	**4** Weight function
1 Simple geometry: flat gap γ=1, round pipe γ=2	$\frac{8}{\lambda}=\mathrm{Re}\int_0^1 \frac{\left(\gamma-\sum_e \Phi_{ew}K_{ew}\right)\cdot\Re_\tau^2}{\gamma\tilde{\rho}\tilde{\nu}_\tau R^{\gamma-1}}dR$, $\frac{1}{St_q}=Pe\tilde{\eta}\int_0^1 \frac{\left(\gamma-\sum_e \Phi_{eh}K_{eh}\right)\cdot\Re_q^2}{\gamma\tilde{\rho}\tilde{k}_T R^{\gamma-1}}dR$	$K_{\rho w}=1-\frac{\int_0^R \tilde{\rho}R_\gamma dR}{\int_0^1 \tilde{\rho}R_\gamma dR}$, $K_{qh}=1-\frac{\int_0^R \tilde{q}_v R_\gamma dR}{\int_0^1 \tilde{q}_v R_\gamma dR}$	$\Re_\sigma=\frac{\int_0^R \rho w^\sigma R_\gamma dR}{\int_0^1 \rho w^\sigma R_\gamma dR}$, σ=0 and $_{(\sigma=\tau)}$ for λ, σ=1 and $_{(\sigma=q)}$ for St_q
2 Concentric annular channel with inner zone *a*=1, sign "-" and outer zone *a*=2, sign "+"	$\frac{8}{\lambda_a}=\mathrm{Re}_a\int_0^1 \frac{\left\{1\mp 2\tilde{\delta}_{wa}\left(1-\sum_e \Phi_{ewa}K_{ewa}\right)\right\}\cdot\Re_{\tau a}^2}{\tilde{\rho}_a\tilde{\nu}_{Ta}Y_{\tau a}}dY$, $\frac{1}{St_{qa}}=Pe_a\tilde{\eta}_a\int_0^1 \frac{\left\{1\mp 2\tilde{\delta}_{qa}\left(1-\sum_e \Phi_{eha}K_{eha}\right)\right\}\cdot\Re_{qa}^2}{\tilde{\rho}_a\tilde{k}_{Ta}Y_{qa}}dY$	$K_{\rho wa}=1-\frac{\int_0^{Y_a}\tilde{\rho}_a Y_{\tau a}\left(1\mp\tilde{\delta}_{wa}\right)dY}{\int_0^1 \tilde{\rho}_a Y_{\tau a}Y_a\left(1\mp\tilde{\delta}_{wa}Y_a\right)dY}$, $K_{qha}=1-\frac{\int_0^{Y_a}\tilde{q}_{va}Y_{qa}\left(1\mp\tilde{\delta}_{ha}\right)dY}{\int_0^1 \tilde{q}_{va}Y_{qa}Y_a\left(1\mp\tilde{\delta}_{ha}Y_a\right)dY}$	$\Re_{\sigma a}=\frac{\int_0^{Y_a}\rho_a w_a^\sigma Y_{Ja}dY}{\int_0^1 \rho_a w_a^\sigma Y_{Ja}dY}$, $Y_{Ja}=1\mp 2\tilde{\delta}_{\varphi a}Y_a$
3 Azimuthal angle segment Δ near of the pin wall number-n	$\frac{8}{\lambda_n}=\frac{\mathrm{Re}_n}{\Delta}\int_0^{\Delta}\int_0^1 \frac{\left\{\tilde{\tau}_{wn}-2\tilde{\delta}_{wn}\left(\tilde{\tau}_{wn}-\sum_e \Phi_{ewn}K_{ewn}\right)\right\}\cdot\Re_{\tau n}^2}{\tilde{\rho}_n\tilde{\nu}_{Tn}Y_{\tau n}}dYd\theta$, $\frac{1}{St_{qn}}=\frac{Pe_n\tilde{\eta}_n}{\Delta}\int_0^{\Delta}\int_0^1 \frac{\left\{\tilde{q}_{wn}-2\tilde{\delta}_{hn}\left(\tilde{q}_{wn}-\sum_e \Phi_{ehn}K_{ehn}\right)\right\}\cdot\Re_{qn}^2}{\tilde{\rho}_n\tilde{k}_{Tn}Y_{qn}}dYd\theta$	$K_{\rho wn}^{\wedge}=1-\frac{\int_0^{\Delta}\int_0^{Y_n}\tilde{\rho}_n Y_{\tau n}\left(1-\tilde{\delta}_{wn}\right)dYd\theta}{\int_0^{\Delta}\int_0^1 \tilde{\rho}_n Y_{\tau n}Y_n\left(1-\tilde{\delta}_{wn}Y_n\right)dYd\theta}$, $K_{qhn}^{\wedge}=1-\frac{\int_0^{\Delta}\int_0^{Y_n}\tilde{q}_{vn}Y_{qn}\left(1-\tilde{\delta}_{hn}\right)dYd\theta}{\int_0^{\Delta}\int_0^1 \tilde{q}_{vn}Y_{qn}Y_n\left(1-\tilde{\delta}_{hn}Y_n\right)dYd\theta}$	$\Re_{\sigma n}=\frac{\int_0^{\Delta}\int_0^{Y_a}\rho_n w_n^\sigma Y_{Jn}dYd\theta}{\int_0^{\Delta}\int_0^1 \rho_n w_n^\sigma Y_{Jn}dYd\theta}$, $Y_{Jn}=1-2\tilde{\delta}_{wn}Y_n$

Tab. 4: Analytical closure relationships for friction factor, heat and mass transfer for pipe/gap, annular and sub-channel geometries

Bubbly upward tube flow at low mass velocities have recorded the occurrence of heterogeneous (saddle-shaped) void fraction profiles and anomalous shear stresses.

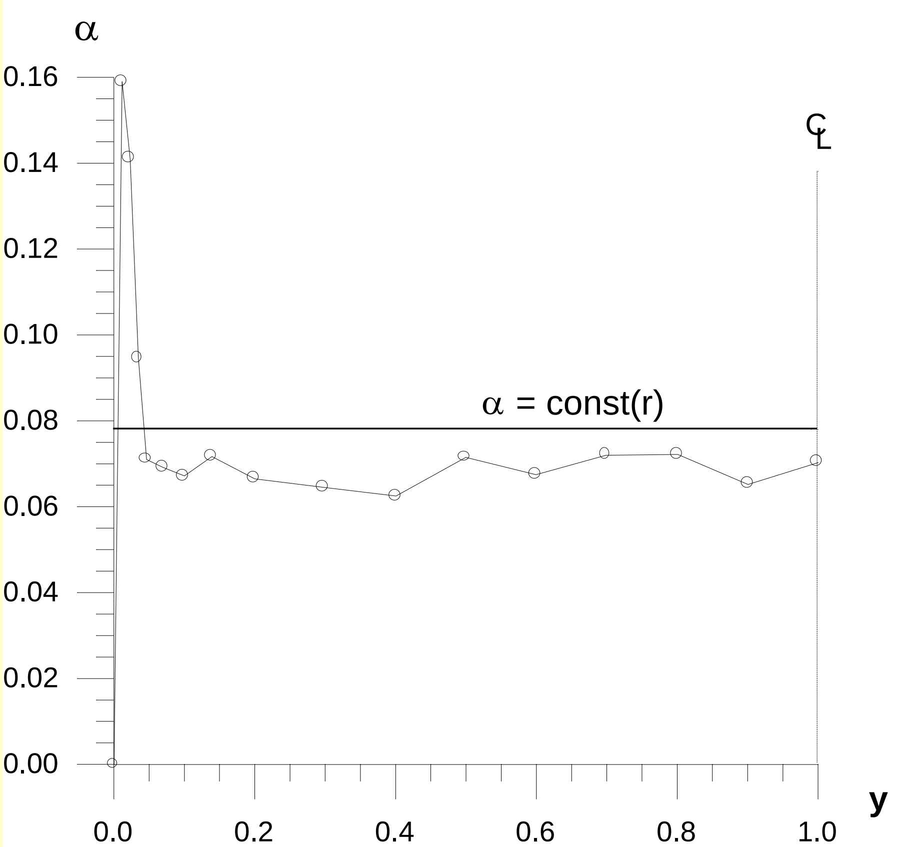


Fig. 1. Bubble saddle-shape void fraction profile for adiabatic two-phase tube flow for inlet condition *Re=19100, β=0.15.* Experiments of Nakoryakov et al., 1981. d_{bubble} = 3 ÷ 5 mm; d_{tube} = 86.4 mm

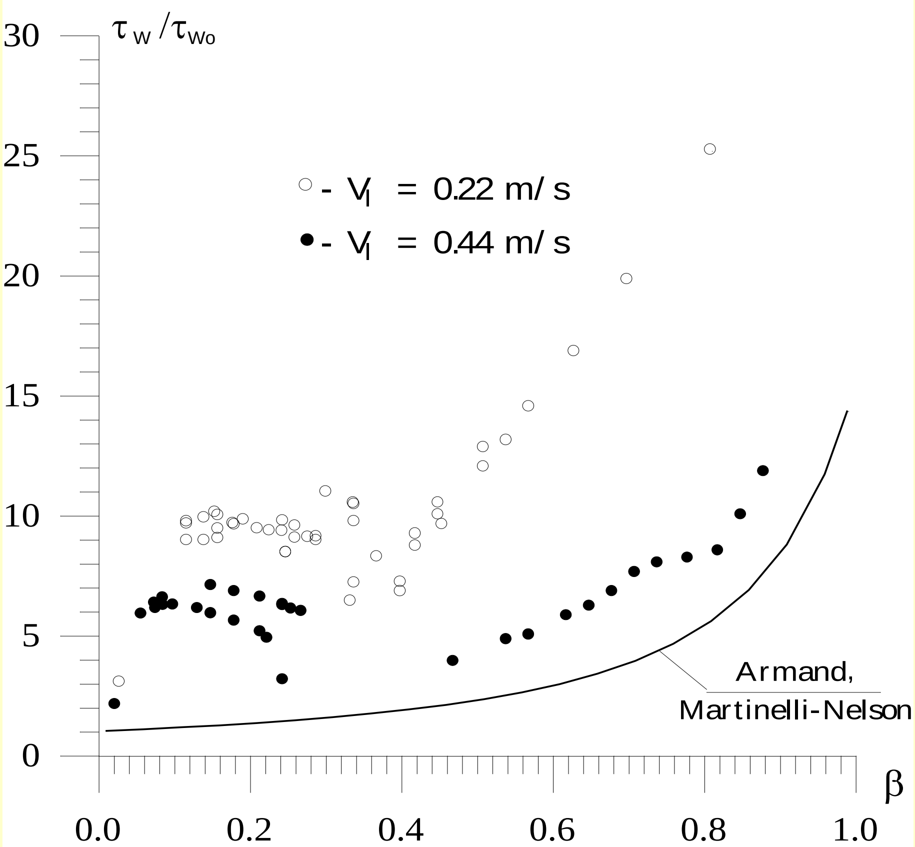


Fig. 2. Wall shear stress ratio (λ/λ_0) vs gas volumetric flow ratio. Anomalous for 0.01< β < 0.2÷0.3. Experiments of Nakoryakov et al., 1981.

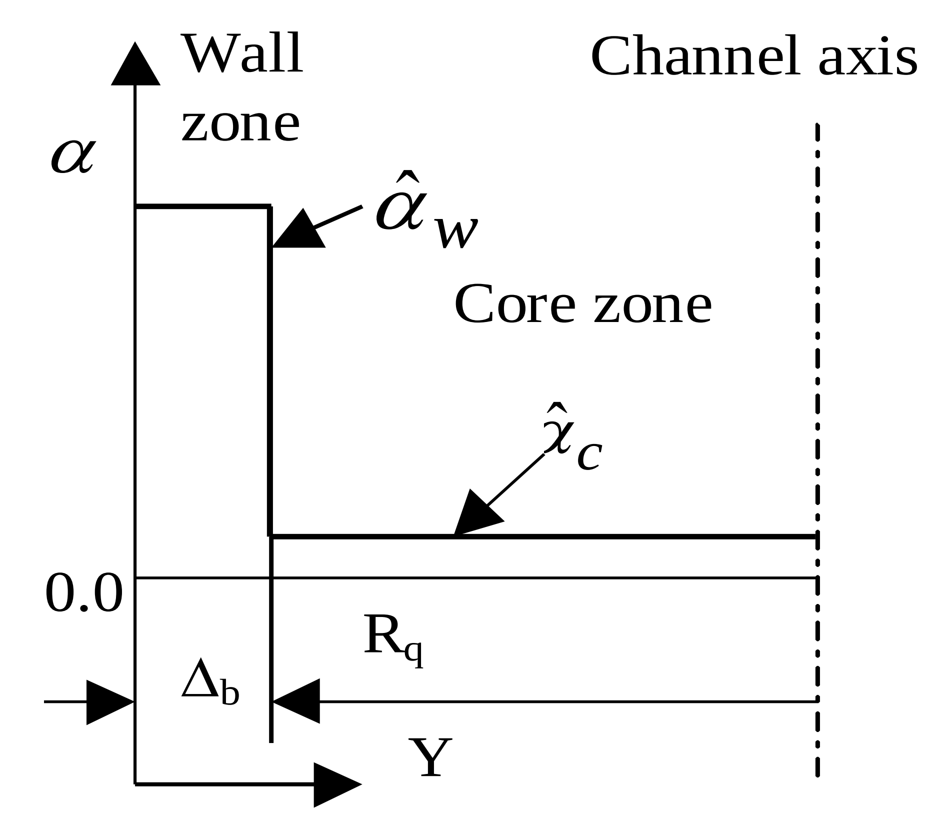


Fig. 6. Geometry and idealization of the saddle-shape bubble flow.

$$\Psi_{\alpha w} = \frac{\lambda}{\lambda_o} = \frac{1}{4}\left(1 + \sqrt{1 + \Phi_{\alpha w} K_{\alpha w}}\right)^2$$

$$\Phi_{\alpha w} = \left(\rho_f - \rho_g\right) / \left(\rho_f \gamma Fr_*\right)$$

$$K_{\alpha w} = C_q \Delta_b \left(\hat{\alpha}_w - \hat{\alpha}_c\right)$$

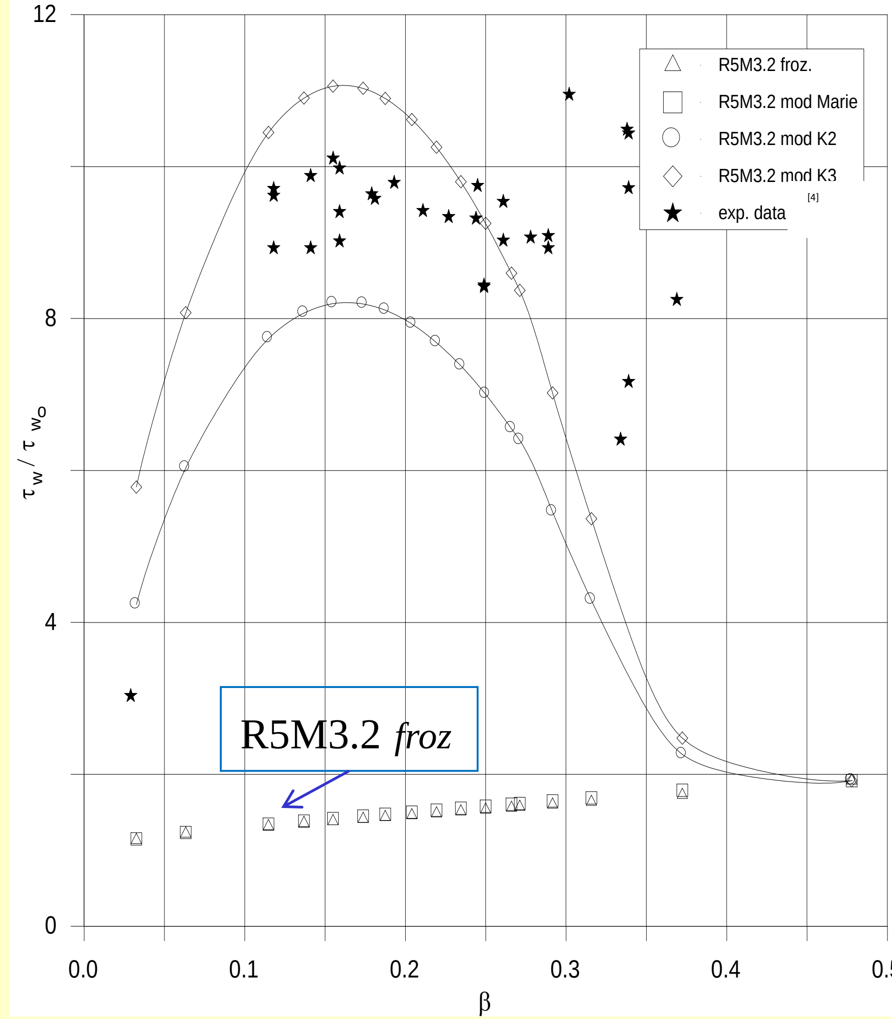


Fig. 7. Wall shear stress ratio vs gas volumetric flow ratio. Re=19100, Nakoryakov et al, 1981.

CONCLUSIONS (Part 2)

A simple and descriptive approach has been proposed to construct generalized quasi-one-dimensional integral Lyon-type relationships for the pipe, annular, and sub-channel wall friction, heat and mass transfer coefficients. The approach is based on: 1) the drift flux model, 2) the boundary layer approximations and 3) a generalized substance transfer notations. The model takes into account both the effect of non-uniform flow profiles as the effect of the geometry (pipe, annular and sub-channel type).

With this approach, one can formulate the integral analytical expressions for the wall friction factor, heat, and mass transfer coefficients to account for the contribution of various complementary effects. These additional effects are heterogeneous profiles of generalized mass forces arising due to the presence of local variable gradients in the non-equilibrium flows. They include not only the density (in the mixed convection), but also other components in the momentum, heat, and mass transfer processes, and their sources and sinks in the channel flow cross section.

Unlike Lyon', Kutateladze-Leont'yev', Petukhov-Popov', Novikov-Voskresensky', and Iannello-Suh-Todreas' relationships, the integral forms deduced are more general and are characterized by an additive form of notation of the effects under consideration. This is significant for the criteria to assess the contribution of the effect in question.

Thank you
for your attention

3D formulation of two-fluid for k phase (or drift flux model (DFM))

$$\frac{\partial \rho_k \varphi_k}{\partial t} + \nabla \rho_k w_k \varphi_k = -\nabla \bar{\bar{J}} + I_v$$

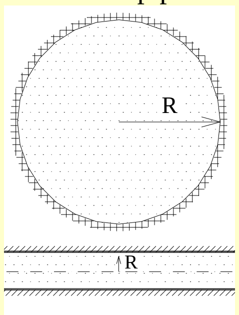
(1)

for slender channel φ_{ki} – potential: α_k, T_k, (h_k), w_k – liquid (or gas) axial velocity. DFM: volume flux density

$$j = \sum_{k=1}^{2} \alpha_k w_k$$

$$k = \begin{cases} 1, \ or\ f - fluid \\ 2, \ or\ g - gas \end{cases}$$

Round pipe



Slot channel

(3)

$$\frac{\partial \langle \rho_k \varphi_k \rangle}{\partial t} + \frac{\partial}{\partial z} \langle \rho_k w_k \varphi_k \rangle = \langle F_w'''(St_k ...) \rangle + \langle \Psi_i \rangle$$

wall losses

$$+ \langle I_{kv} \rangle$$

source (or sink)

Initial and boundary conditions of the channel

Space averaging over a simple cross-section $\boldsymbol{A_p}$

(2)

$$C_{ki} = \frac{A_p \cdot \int_0^{A_p} \varphi_{ki} \cdot j dA}{\int_0^{A_p} \varphi_{ki} dA \cdot \int_0^{A_p} j dA} \neq 1$$

$$A_p = \begin{cases} \pi R^2 - round\ \ pipe \\ RL - slot\ channel \end{cases}$$

Space averaging over a subchannel cross-section $\boldsymbol{A_n}$ or an annular (sector) area $\boldsymbol{A_\Delta}$

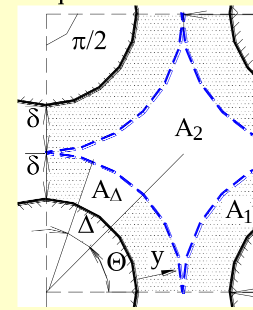
(4)

$$C_{kin} = \frac{A_n \cdot \int_0^{A_n} \varphi_{ki} \cdot jdA}{\int_0^{A_n} \varphi_{ki} dA \cdot \int_0^{A_n} jdA} \neq 1$$

$$A_n = A_1 + A_2; \ \tilde{A}_1 = A_1 / A_n$$

$$\tilde{A}_1 = \frac{\tilde{\delta}(2 - \tilde{\delta})}{E\tilde{s}^2 - (1 - \tilde{\delta})^2}; \ E = \frac{4}{\pi}$$

Square subchannel



$d_1 = 2r_1$; $r_0 = r_1 + \delta$; $s = 2r_0$

$\tilde{\delta} = \delta / r_0; \tilde{s} = s / d_1$

(5)

$$\frac{\partial \langle \rho_k \varphi_k \rangle_n}{\partial t} + \frac{\partial}{\partial z} \langle \rho_k w_k \varphi_k \rangle_n = \langle F_w'''(St_k ...) \rangle_n + \langle \Psi_{in} \rangle$$

wall losses

$$+ \langle I_{kv} \rangle_n + \beta_{kn}$$

source *inter – subchannel exchange*

Initial and boundary conditions of the subchannel

Radial profile effects of the substance fluxes $\tilde{J}_{rz}$ and sources (or sinks) $\boldsymbol{I_{rz}}$

(7)

$$\frac{1}{Nu_k} = \frac{1}{\tilde{\eta}\, St_k Pe_k} = \int_0^1 \Re \left(\int_0^R \frac{\tilde{J}_k dR}{\tilde{\rho} \varepsilon_{kT}} \right) R_\gamma dR$$

$\Re = f(\rho w)$ – weight function for simple channel, see bottom of Table 3.

3D closure relationships:

1) for interface (i) Ψ_i $\Gamma_{\mp}$, $(M_i\, q_i''')$ and

2) for turbulent substance transfer ε_{kT}

Transversal profiles effect of the various components

(6)

$$\tilde{J} = \int_0^Y \left(\frac{\partial \rho \varphi}{\partial t} + \nabla \rho w \varphi - I_v \right) dy$$

Transversal distribution effects of the substance fluxes $\tilde{J}_{r\theta z}$ and sources

(8)

$$\frac{1}{\hat{Nu}_{kn}} = \frac{1}{\hat{St}_{kn} \hat{Pe}_{kn} \hat{\eta}_{kn}} = \int_0^{A_n} \Re_n \left(\int_0^{A_Y} \frac{\tilde{J}_{kn} dY}{\tilde{\rho} \varepsilon_{kTn}} \right) d\tilde{A}$$

$\Re_n = f(\rho w)$ – weight function for compound (sub)channel

Figure 1. Space averaging scheme of the local parameters over the simple channel geometry and the subchannel geometry cross-section.

1	2 e	3 Component	4 Averaged transfer components $\Phi_{e\varphi}$, $\Phi_{e\varphi a}$, $\Phi_{e\varphi n}$	5 Form-factors for the variable profile $K_{e\varphi}$, $K_{e\varphi a}$, $K_{e\varphi n}$
P	1	Source/ sink	$\Phi_{v\varphi}=\int_0^1 \tilde{I}_{v\varphi} R_\gamma dR, \quad R_\gamma = \gamma R^{\gamma-1},$ (1)	$K_{v\varphi} = 1-\int_0^R \tilde{I}_{v\varphi} R_\gamma dR \Big/ \left(\Phi_{v\varphi}\Re_J\right)$ (2)
I	2*	Axial gradient	$\Phi_{z\varphi}=\int_0^1 \tilde{\rho} w^+ \frac{\partial\varphi^+}{\partial Z} R_\gamma dR$ (3)	$K_{z\varphi} = 1-\int_0^R \tilde{\rho} w^+ \frac{\partial\varphi^+}{\partial Z} R_\gamma dR \Big/ \left(\Phi_{z\varphi}\Re_J\right)$ (4)
P	3	Radial gradient	$\Phi_{y\varphi} = \int_0^1 \tilde{\rho} v^+ \frac{\partial\varphi^+}{\partial R} R_\gamma dR$ (5)	$K_{y\varphi} = 1-\int_0^R \tilde{\rho} v^+ \frac{\partial\varphi^+}{\partial R} R_\gamma dR \Big/ \left(\Phi_{y\varphi}\Re_J\right)$ (6)
E	4	Temporal acceleration	$\Phi_{\omega\varphi} = \frac{1}{Sr_*}\int_0^1 \tilde{\rho} \frac{\partial\varphi^+}{\partial\tilde{t}} R_\gamma dR$ (7)	$K_{\omega\varphi} = 1-\frac{1}{Sr_*}\int_0^R \tilde{\rho} \frac{\partial\varphi^+}{\partial\tilde{t}} R_\gamma dR \Big/ \left(\Phi_{\omega\varphi}\Re_J\right)$ (8)
A N	1	Source/ sink	$\Phi_{v\varphi a}=\int_0^1 \tilde{I}_{v\varphi a} Y_{Ja} dY,$ where $Y_{Ja} = 1\mp 2\tilde{\delta}_{Ja} Y_a$ (1)	$K_{v\varphi a} = 1-\int_0^{Y_a} \tilde{I}_{v\varphi a} Y_{Ja} dY \Big/ \left(\Phi_{v\varphi a}\Re_{Ja}\right)$ (2)
N U	2*	Axial gradient	$\Phi_{z\varphi a}=\int_0^1 \tilde{\rho}_a w_a^+ \frac{\partial\varphi_a^+}{\partial Z} Y_{Ja} dY$ (3)	$K_{z\varphi a} = 1-\int_0^{Y_a} \tilde{\rho}_a w_a^+ \frac{\partial\varphi_a^+}{\partial Z} Y_{Ja} dY \Big/ \left(\Phi_{z\varphi a}\Re_{Ja}\right)$ (4)
L A	3	Radial gradient	$\Phi_{y\varphi a} = \int_0^1 \tilde{\rho}_a v_a^+ \frac{\partial\varphi_a^+}{\partial Y} Y_{Ja} dY$ (5)	$K_{y\varphi a} = 1-\int_0^{Y_a} \tilde{\rho}_a v_a^+ \frac{\partial\varphi_a^+}{\partial Y} Y_{Ja} dY \Big/ \left(\Phi_{y\varphi a}\Re_{Ja}\right)$ (6)
R	4	Temporal acceleration	$\Phi_{\omega\varphi a} = \frac{1}{Sr_{*a}}\int_0^1 \tilde{\rho}_a \frac{\partial\varphi_a}{\partial\tilde{t}} Y_{Ja} dY$ (7)	$K_{\omega\varphi a} = 1-\frac{1}{Sr_{*a}}\int_0^{Y_a} \tilde{\rho}_a \frac{\partial\varphi_a^+}{\partial\tilde{t}} Y_{Ja} dY \Big/ \left(\Phi_{\omega\varphi a}\Re_{Ja}\right)$ (8)

Tab. 3: Definitions for $\Phi_{e\varphi}$ components' average values and form-factors $K_{e\varphi}$ in equations of (φ) substances transport for pipe/gap, annular and sub-channel geometries. 2^* - absent for heat and mass fluxes, using $\partial\varphi^+/\partial Z$

S U	1	Source/ sink	$\Phi^{\wedge}_{v\varphi n}=\frac{1}{\Delta}\int\limits_0^{\Delta}\int\limits_0^{1}\tilde{I}_{v\varphi n}Y_n\,dY\,d\theta\,,$ (1) $Y_n=1-2\tilde{\delta}_n Y'$ (1)	$K^{\wedge}_{v\varphi n}=1-\frac{1}{\Delta}\int\limits_0^{\Delta}\int\limits_0^{Y_n}\tilde{I}_{v\varphi n}Y_n\,dY\,d\theta \Big/ \left(\Phi^{\wedge}_{v\varphi n}\Re^{\wedge}_{Jn}\right)$ (2)
B	2*	Axial Gradient	$\Phi^{\wedge}_{z\varphi n}=\frac{1}{\Delta}\int\limits_0^{\Delta}\int\limits_0^{1}\tilde{\rho}_n w_n^{+}\frac{\partial\varphi_n^{+}}{\partial Z}Y_n\,dY\,d\theta$ (3)	$K^{\wedge}_{z\varphi n}=1-\frac{1}{\Delta}\int\limits_0^{\Delta}\int\limits_0^{Y_n}\tilde{\rho}_n w_n^{+}\frac{\partial\varphi_n^{+}}{\partial Z}Y_n\,dY\,d\theta \Big/ \left(\Phi^{\wedge}_{z\varphi n}\Re^{\wedge}_{Jn}\right)$ (4)
C H	3	Radial gradient	$\Phi^{\wedge}_{y\varphi n}=\frac{1}{\Delta}\int\limits_0^{\Delta}\int\limits_0^{1}\tilde{\rho}_n v_n^{+}\frac{\partial\varphi_n^{+}}{\partial Y}Y_n\,dY\,d\theta$ (5)	$K^{\wedge}_{y\varphi n}=1-\frac{1}{\Delta}\int\limits_0^{\Delta}\int\limits_0^{Y_n}\tilde{\rho}_n v_n^{+}\frac{\partial\varphi_n^{+}}{\partial Y}Y_n\,dY\,d\theta \Big/ \left(\Phi^{\wedge}_{y\varphi n}\Re^{\wedge}_{Jn}\right)$ (6)
A N	4	Temporal acceleration	$\Phi^{\wedge}_{\omega\varphi n}=\frac{1}{Sr_{*n}\Delta}\int\limits_0^{\Delta}\int\limits_0^{1}\tilde{\rho}_n\frac{\partial\varphi_n^{+}}{\partial\tilde{t}}Y_n\,dY\,d\theta$ (7)	$K^{\wedge}_{\omega\varphi n}=1-\frac{1}{Sr_{*n}\Delta}\int\limits_0^{\Delta}\int\limits_0^{Y_n}\tilde{\rho}_n\frac{\partial\varphi_n^{+}}{\partial\tilde{t}}Y_n\,dY\,d\theta \Big/ \left(\Phi^{\wedge}_{\omega\varphi n}\Re^{\wedge}_{Jn}\right)$ (8)
N A	5	Azimuthal gradient	$\Phi^{\wedge}_{\theta\varphi n}=\frac{1}{\Delta}\int\limits_0^{\Delta}\int\limits_0^{1}\tilde{\rho}v_{\theta}{}^{+}\frac{\partial\varphi^{+}}{\partial\theta}\,dY\,d\theta \Big/ \Re^{\wedge}_{Jn}$ (9)	$K^{\wedge}_{\theta\varphi n}=1-\frac{1}{\Delta}\int\limits_{\theta}^{\Delta}\int\limits_0^{Y_n}\tilde{\rho}v_{\theta}{}^{+}\frac{\partial\varphi^{+}}{\partial\theta}\,dY\,d\theta \Big/ \left(\Phi^{\wedge}_{\theta\varphi n}\Re^{\wedge}_{Jn}\right)$ (10)
L	6	Azimuthal gradient	$\Phi^{\wedge}_{\theta Jn}=\frac{1}{\Delta}\int\limits_0^{\Delta}\int\limits_0^{1}\frac{\partial\tilde{\tilde{J}}_{\theta}}{\partial\theta}\,dY\,d\theta \Big/ \Re^{\wedge}_{Jn}$ (11)	$K^{\wedge}_{\theta Jn}=1-\frac{1}{\Delta}\int\limits_{\theta}^{\Delta}\int\limits_0^{Y_n}\frac{\partial\tilde{\tilde{J}}_{\theta}}{\partial\theta}\,dY\,d\theta \Big/ \left(\Phi^{\wedge}_{\theta Jn}\Re^{\wedge}_{Jn}\right)$ (12)

Tab. 3 (continued): Definitions for $\Phi_{e\varphi}$ components' average values and form-factors $K_{e\varphi}$ in equations of (φ) substances transport for pipe/gap, annular and sub-channel geometries. 2^* - absent for heat and mass fluxes, using $\partial\varphi^+/\partial Z$