The Validation of Open-source Code Gerris on the Problems of Hydrodynamic Instabilities

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Why we choose the simulation of instabilities for validation?

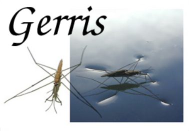
- Hydrodynamic instabilities are driven mechanism of the appearance of some effects.
- Correct numerical simulation can't be possible without correct simulations of instabilities.
- There are some analytical solutions for hydrodynamic instabilities.

Main types of hydrodynamic instabilities

- Rayleigh — Taylor (penetration of heavy liquid to light one);
- Kelvin — Helmholtz (increasing of wave amplitude in case of two-phase flow with different velocities);
- Plateau — Rayleigh (breakup of liquid jet by influence of surface forces).

Gerris

Gerris — open-source code for the numerical solution of the hydrodynamic problems (particularly for free-surface flows).



Main features

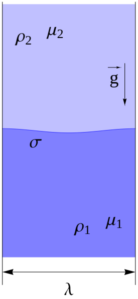
- Volume of Fluid scheme for interfacial flows.
- Level-set function is uses for interface detecting.
- Accurate surface tension model.
- Adaptive mesh refinement: the resolution is adapted dynamically to the features of the flow.
- Portable parallel support using the MPI library, dynamic load-balancing.

Rayleigh — Taylor instability

It is observed during penetration of the light liquid in the heavy one.

Instability conditions:

1. Initial perturbation $y = \alpha_0 \cos(kx)$ with amplitude $\alpha_0 > 0$, $\alpha_0 \ll \lambda$.
2. Surface tension coefficient: $\sigma < \sigma_c$, $\sigma_c = \dfrac{\Delta\rho g}{k^2}$.
3. Coefficients of dynamic viscosities: $\mu_1 = \mu_2 = \mu$.



Linear theory:

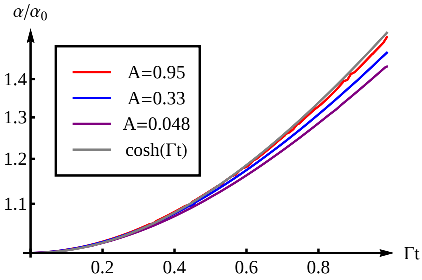
If conditions are satisfied then amplitude of instability grows by law:

$$\alpha(t) = \alpha_0 \cosh(\Gamma t),$$

$\Gamma = \sqrt{kg\left(A - \dfrac{k^2\sigma}{g(\rho_1+\rho_2)}\right)}$ — growth rate, $[\Gamma] = 1/s$;

$A = \dfrac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$ — dimensionless Atwood number.

Simulation results. Mesh convergence



Case parameters

- $\rho_2 = 1.255$ kg/m^3, $\rho_1 = \rho_1 \frac{1-A}{1+A}$;
- $\mu = 3.13 \cdot 10^{-3}$ kg/(m $\cdot$ s);
- $\sigma = 0.0$ N/m; $g = 9.81$ m/s^2;
- $\lambda = 1$ m, $\alpha_0 = 0.05$ m;
- Mesh: $2^5, 2^6, 2^7, 2^8, 2^9, 2^{10}$ cells per wave length.

Error as compared to the reference results (for mesh 2^{10}).

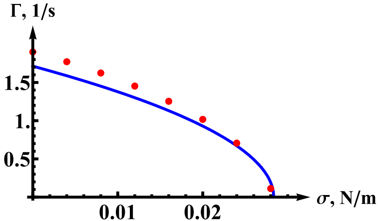
A	$\|\|f_{10} - f_5\|\|_{L_2}$	$\|\|f_{10} - f_6\|\|_{L_2}$	$\|\|f_{10} - f_7\|\|_{L_2}$	$\|\|f_{10} - f_8\|\|_{L_2}$	$\|\|f_{10} - f_9\|\|_{L_2}$
0.95	0.00145	0.00242	0.00215	0.00225	0.00153
0.33	0.00790	0.00273	0.00070	0.00050	0.00044
0.048	0.00669	0.00193	0.00045	0.00013	0.00009

Instability region

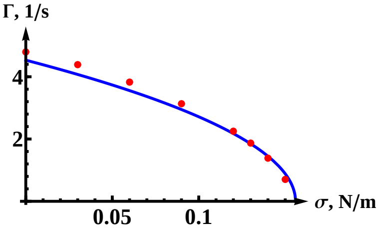
Dependence of the growth rate

Dependence of the growth rate on the Atwood number and the surface tension coefficient (for previous case):

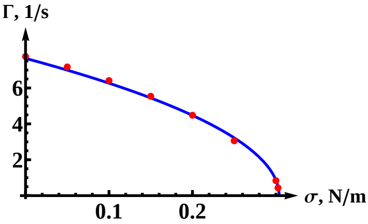
$$\Gamma = 7.85\sqrt{A - 1.60\,(1+A)\,\sigma}.$$



A = 0.048

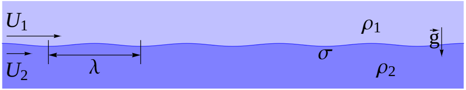


A = 0.33



A = 0.95

Kelvin — Helmholtz instability



It is observed at the interface of the two liquids when it move with different velocities.

Instability conditions:

1. Initial perturbation $y = \alpha_0 \cos(kx)$ with amplitude $\alpha_0 > 0$, $\alpha_0 \ll \lambda$.
2. Condition for velocity difference: $\Delta U^2 > \dfrac{(\rho_1 + \rho_2)(g\Delta\rho + k^2\sigma)}{k\rho_1\rho_2} = \Delta U_{cr}^2$.

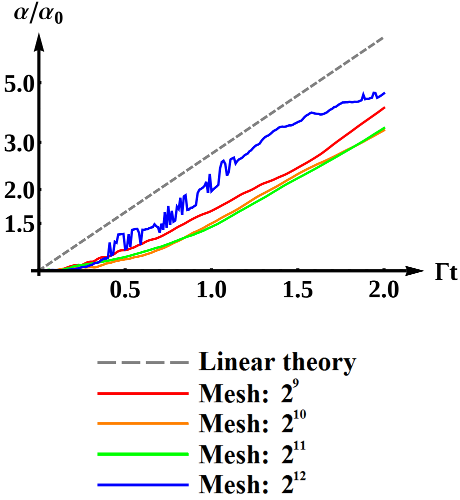
Linear theory:

If conditions are satisfied then amplitude of instability grows by law:

$$\alpha\left(t\right) = \alpha_0 \exp\left(\Gamma t\right),$$

$\Gamma = \left(\dfrac{k^2 \Delta U^2 \rho_1\rho_2}{(\rho_1+\rho_2)^2} - \dfrac{k^3\sigma + kg\Delta\rho}{\rho_1+\rho_2}\right)^{1/2}$ — growth rate, $[\Gamma] = 1/s$.

Mesh Convergence

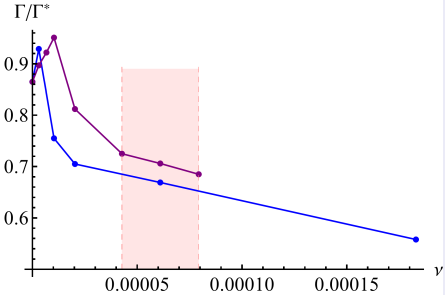


Case parameters

- $\lambda = 0.1$ m,
  $\alpha_0 = 0.0008$ m;
- $\rho_1 = 960$ kg/m^3,
  $\rho_2 = 998$ kg/m^3;
- $\Delta U = 3.0$ m/s;
- $\sigma = 15.0$ N/m;
  $g = 400.0$ m/s^2;
- Mesh: $2^9, 2^{10}, 2^{11}, 2^{12}$ cells per wave length or 8, 16, 32 and 64 cells per wave height.

Dependence of the growth rate from viscosity

Small non-physical waves can be appear on the interface because of initial imbalance of forces caused by interface perturbation. This negative effect can be decreased by adding of viscous forces near the two-phase interface. Artificial kinematic viscosity coefficient depends on cell size proportionally: $\nu \sim \Delta x$.



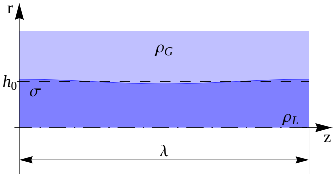
Blue line — viscous term is added in all region, purple line — only near the interface ($\alpha \in (0,\, 1)$).

Case Parameters:

- $\lambda = 0.1$ m, $\alpha_0 = 0.0008$ m;
- $\rho_1 = \rho_2 = 960$ kg/m^3;
- $\Delta U = 3.0$ m/s;
- $\sigma = 18.51$ N/m; $g = 0.0$ m/s^2;
- Mesh: 2^{12} cells per wave length or 64 cells per wave height.

Rayleigh — Plateau instability. Linear theory

It is consist in breakup of the liquid cylinder into droplets because of surface tension.



Instability conditions:

1. Initial perturbation $y = \alpha_0 \cos(kx)$ with amplitude $\alpha_0 > 0$, $\alpha_0 \ll \lambda$.
2. Condition for densities: $\rho_G \ll \rho_L$.
3. Condition for wave length: $\lambda > 2\pi h_0$.

Linear theory:

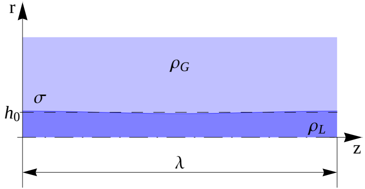
If conditions are satisfied then **radius of cylinder** grows by law:

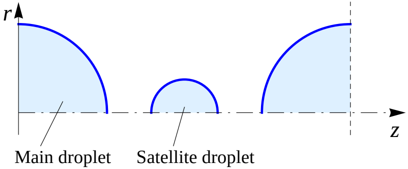
$$h(z,\ t) = h_0 + \varepsilon(t)\cos(kz),\ \varepsilon\left(t\right) = \varepsilon_0 \exp\left(\Gamma t\right),$$

$$\Gamma = \frac{\sigma}{\rho_L h_0^3} kh_0 \left(1 - (kh_0)^2\right) \frac{I_1(kh_0)}{I_0(kh_0)}$$ — growth rate, $[\Gamma] = 1/s$.

Time of breakup can be found as $t_b = \dfrac{\ln\left(h_0/\varepsilon_0\right)}{\Gamma}$.

Rayleigh — Plateau instability. Non-linear theory





Non-linear theory:

Let the instability amplitude depends on initial amplitude as follows:

$$\varepsilon(z,\ t) = \sum_{n=1}^{3} \varepsilon_0^n K_n(z,\ t),$$

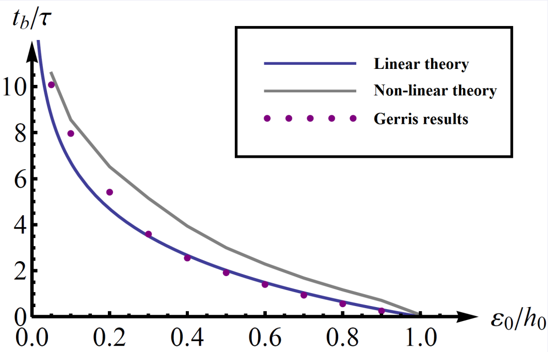
where $K_n(z,\ t)$ are coefficients.
Then **break time** can be founded from condition

$$t_b : \max_{z \in (0,\lambda)} \varepsilon(z,\ t_b) = h_0.$$

Comparison with linear and non-linear theories

Case parameters

- $\lambda = 1.79$ m, $h_0 = 0.2$ m;
- $\rho_L = 998\,\mathrm{kg/m^3}$, $\rho_G = 1.2\,\mathrm{kg/m^3}$;
- $\sigma = 0.72$ N/m;
- Mesh: 57 cells per h_0.

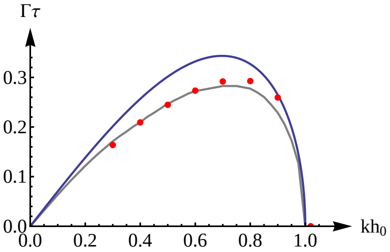


Dimensionless breakup time for different initial amplitude

Comparison with linear and non-linear theories

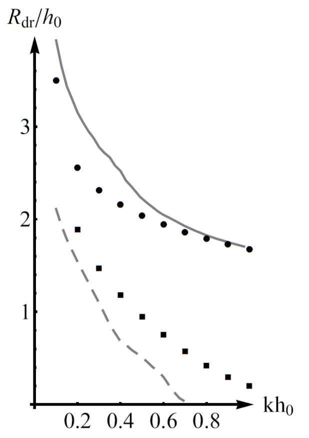
Case parameters

- $h_0 = 0.2\ m$;
- $\rho_L = 998\,\mathrm{kg/m^3}$, $\rho_G = 1.2\,\mathrm{kg/m^3}$;
- $\sigma = 0.72\ \mathrm{N/m}$;
- Mesh: about 25 cells per ε_0.



Blue line — linear theory, gray line — non-linear theory, points — Gerris results.

Size of droplets after breakup



Solid line — size of main droplet, dashed line — size of satellite droplet. Points — results of Gerris simulation.

Conclusion

Rayleigh — Taylor instability

- If Atwood number is close to 1 (big difference between densities) then mesh convergence doesn't observed;
- If Atwood number is close to 0 (fluids with equal densities) then amplitude of instability grows faster then theory predicts.

Kelvin — Helmholtz instability

- Problem statement follows to forces imbalance, additional viscous term near the interface must be entered to balance of forces in initial time;
- Question of the mesh convergence should be investigate in case of adding viscous term.

Plateau — Rayleigh instability

- For the dimensionless wave number $kh_0 < 0.7$ growth rate of instability from simulations agrees with non-linear theory;
- For the dimensionless wave number $kh_0 > 0.7$ jet breaks up to two droplets as predicts by theory.