Software for Wavelet-based Adaptive Grids

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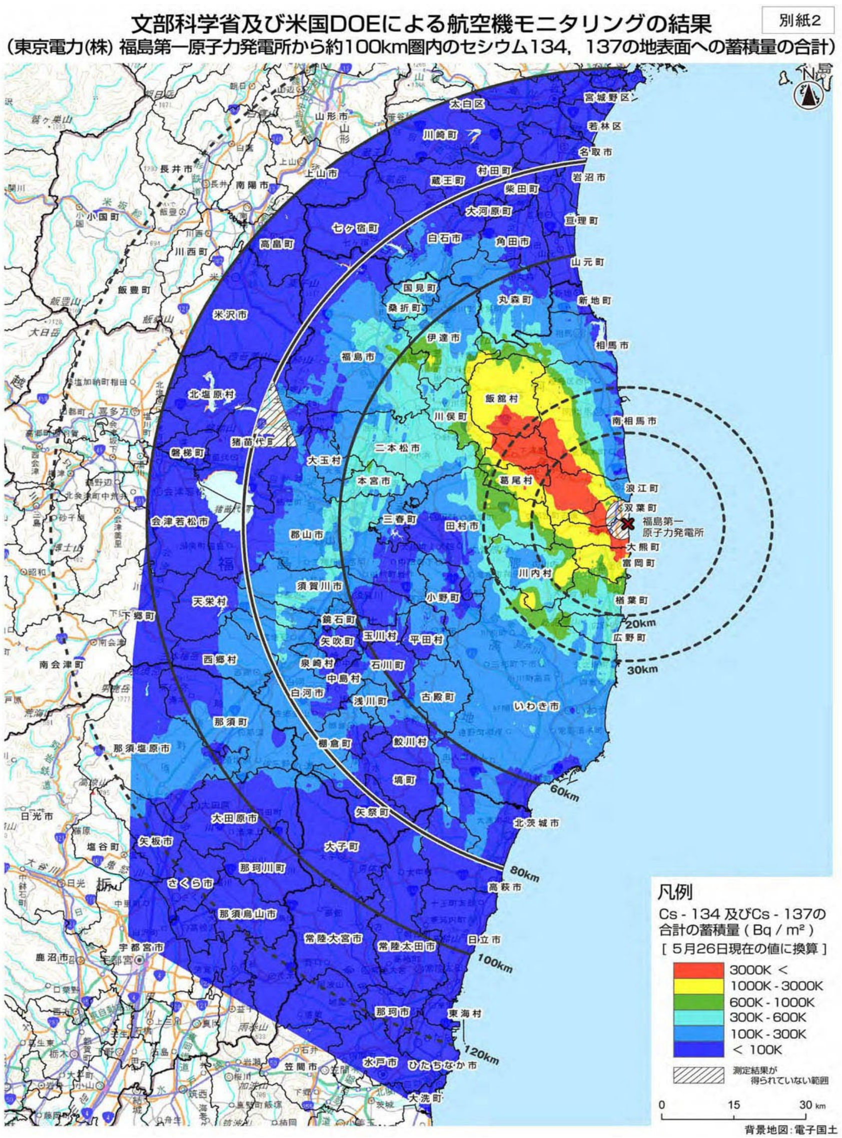
Motivation

- Global atmospheric transport is a very complex multiscale phenomenon. Significant spatial scales of the global atmospheric flow range from small scale convection (~100m or less) to large scale circulation (~1000km). Temporal chemical scales range from fractions of a second to a year or longer.

- However, typical spatial resolution currently used for numerical modeling of atmospheric transport is 0.5º-4º. This relatively crude resolution results in significant numerical diffusion. As a result, numerical error may exceed 90%.

- Therefore, it is necessary to develop a new multiscale numerical technique that allows us to resolve all important spatial scales with presently available computational resources and, hence, produces much more accurate solution than conventional non-adaptive numerical algorithms.
- Wavelet-based Adaptive Mesh Refinement (WAMR)!

Fukushima I Nuclear Disaster
March 11, 2011



Deposition of Cesium on May 26, 2011
(Japan Science Ministry)

Released radioactive materials:
1) Iodine-131,
2) Caesium-134,
3) Caesium-137.

The radioactive isotopes reached
1) the east coast of Russia
in 3 days
2) the west coast of the USA
in 5 days.

Currently Developed Algorithms

Adaptive Mesh Refinement

Block-based grid

PARAMESH

AMRCLAW

AMROC

AMR++

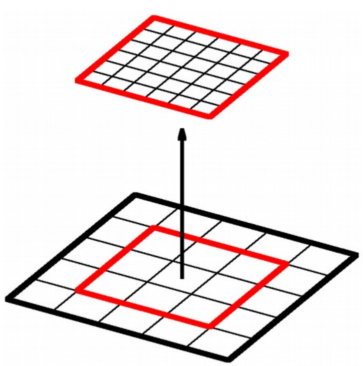
Element-based grid

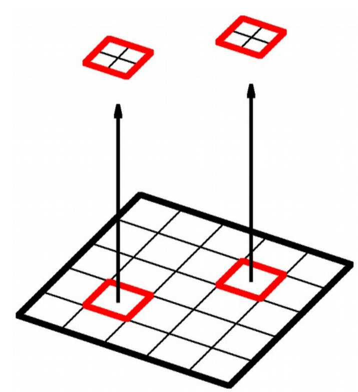
Cell-based grid

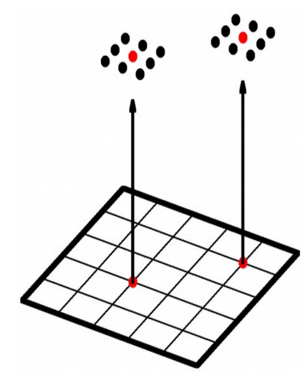
Gerris

Vertex-based grid

WAMR







Advection-Diffusion-Reaction Equation

$$\frac{\partial c_i}{\partial t} + \underbrace{\boldsymbol{u} \cdot \nabla c_i}_{\text{advection}} = \underbrace{\nabla \cdot D \nabla c_i}_{\text{diffusion}} + \omega_i, \quad i = 1, \ldots, N$$

reaction rate

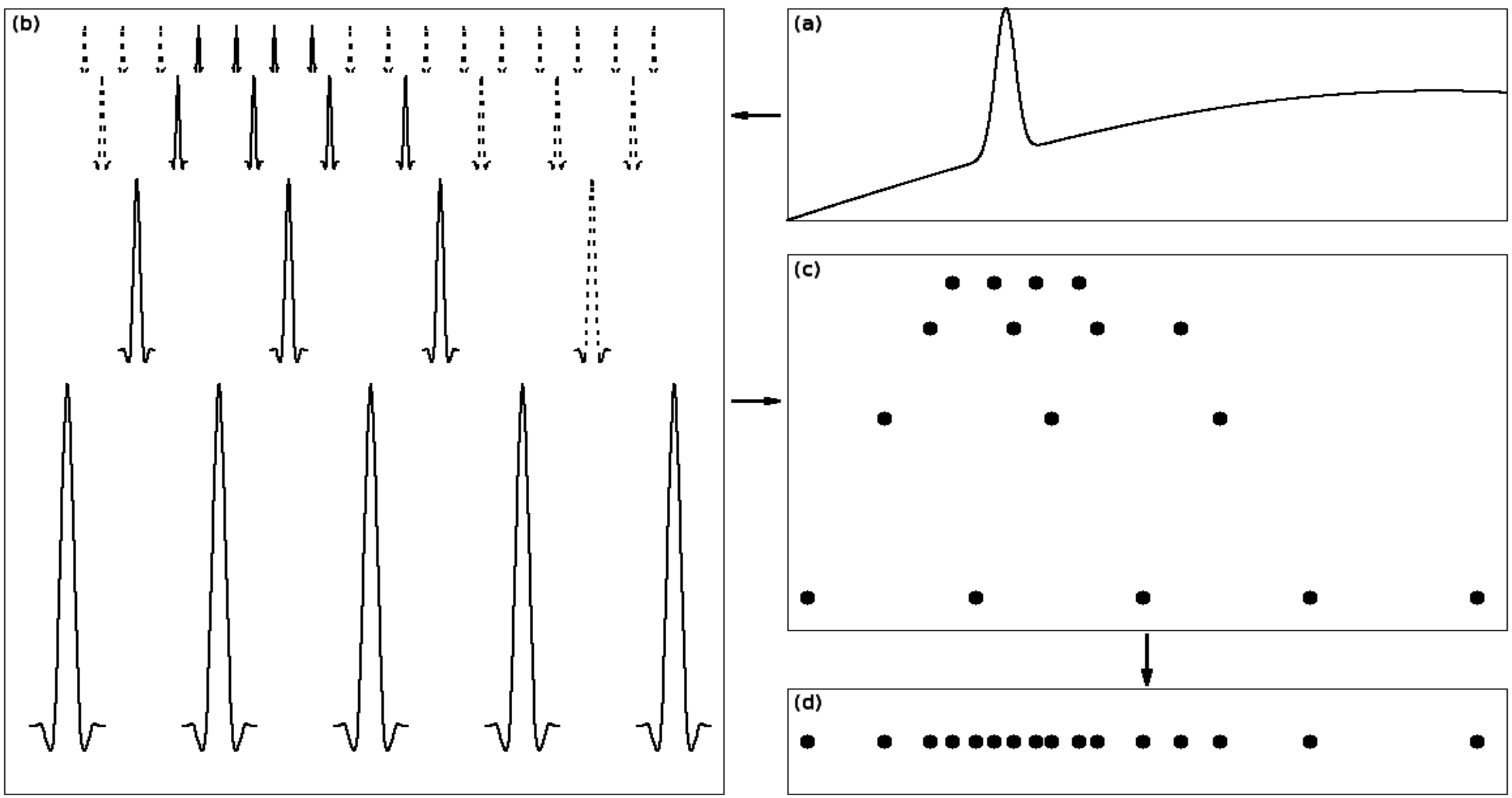
Net chemical production rate is defined as

$$\omega_i = \sum_{l=1}^{N} k_l^i c_l + \sum_{l,j=1}^{N} k_{lj}^i c_l c_j,$$

The system of equations is extremely stiff, nonlinear and involves a large number of species. Typical simulations involve ~ 100 species and ~1000 chemical reactions on different scales: from local to intercontinental – in space, from seconds to days – in time.

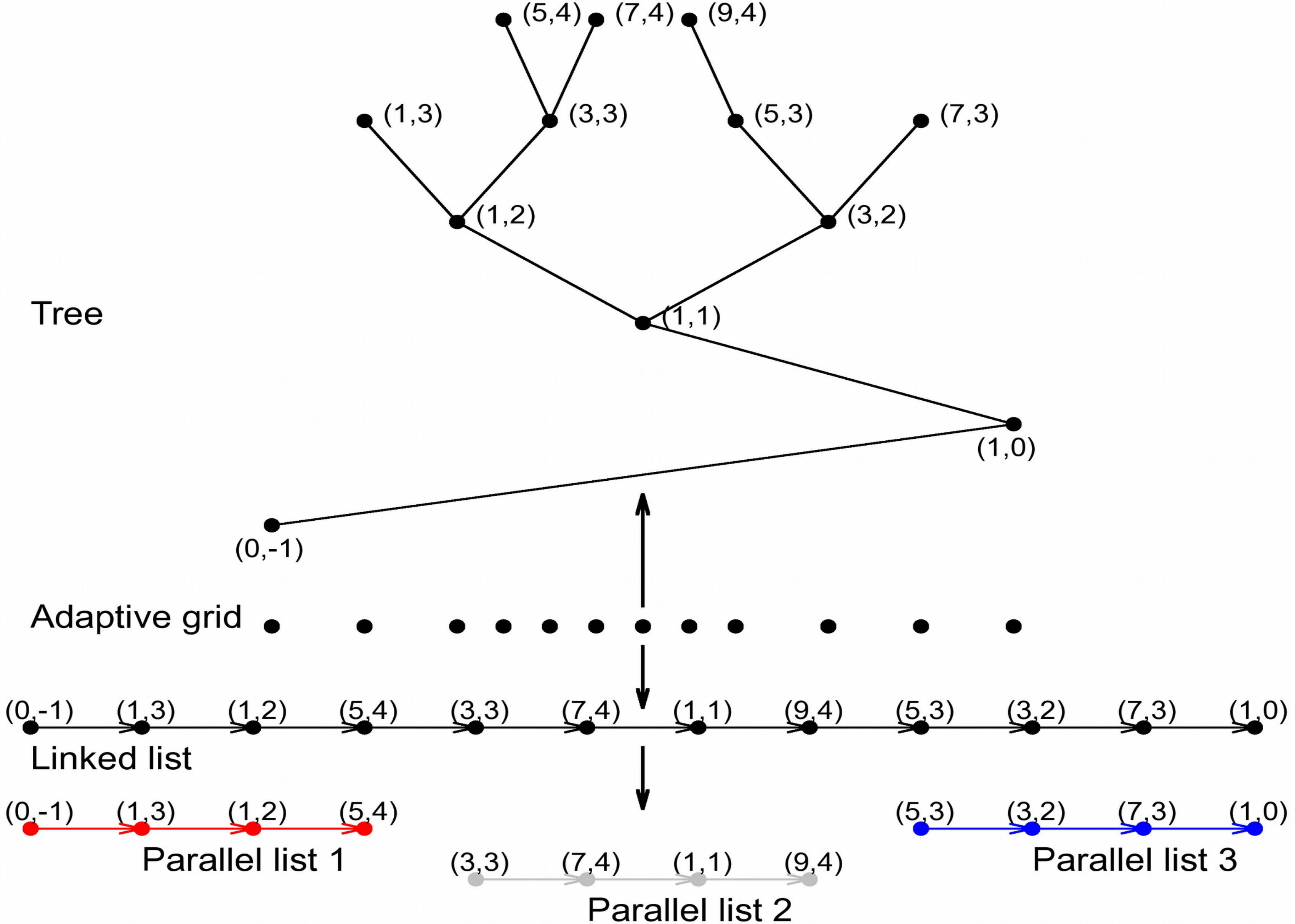
Adaptive Wavelet Basis and Grid

$$c = \sum_k b_k \varphi_k + \sum_{l=0}^{\infty} \sum_k d_{l,k} \psi_{l,k} \longrightarrow P^{\varepsilon} c = \sum_k b_k \varphi_k + \sum_{l=0}^{J} \sum_{|d|>\varepsilon} d_{l,k} \psi_{l,k}$$



Sparse wavelet representation

Adaptive Grid Structure



Advection Equation for an Inert Plume

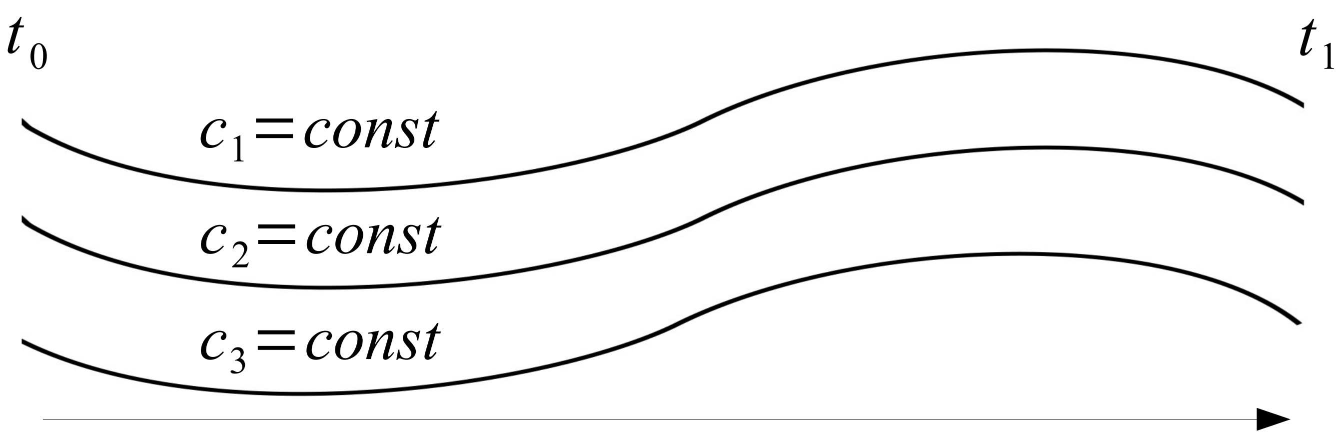
$$\boxed{\frac{\partial c}{\partial t} + \boldsymbol{u} \cdot \nabla c = 0}$$

$$\downarrow$$

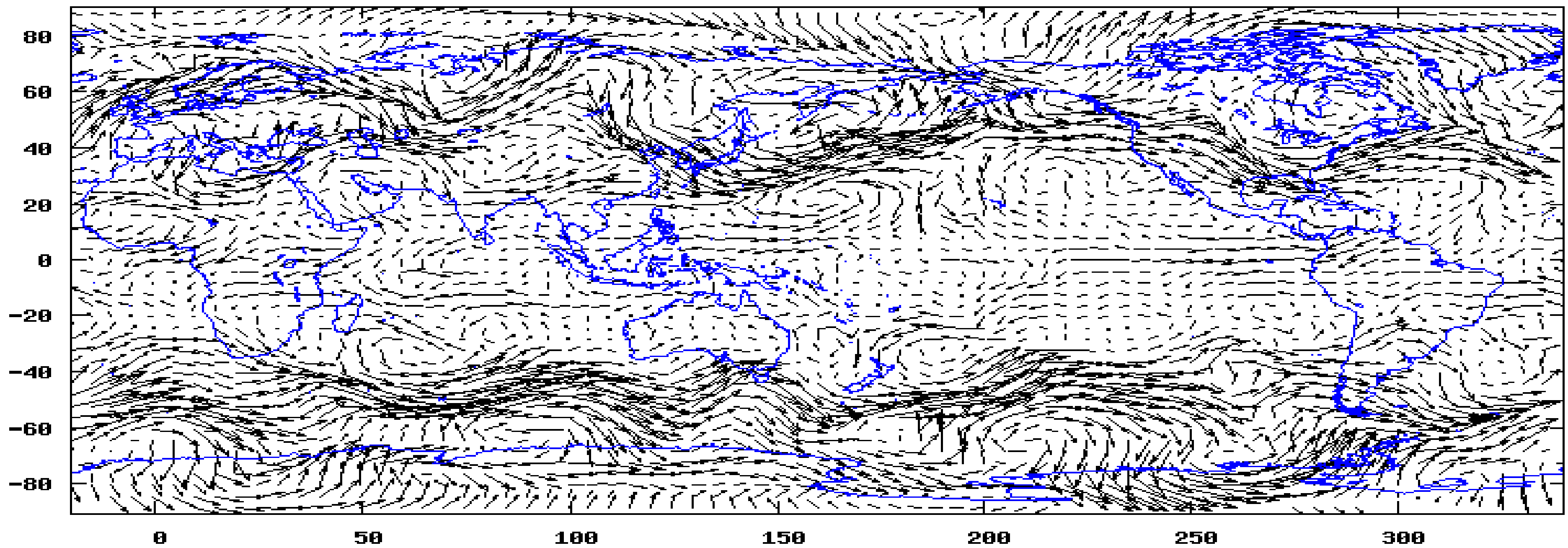
$$\boxed{\frac{dc}{dt} = 0}$$

$$\downarrow$$

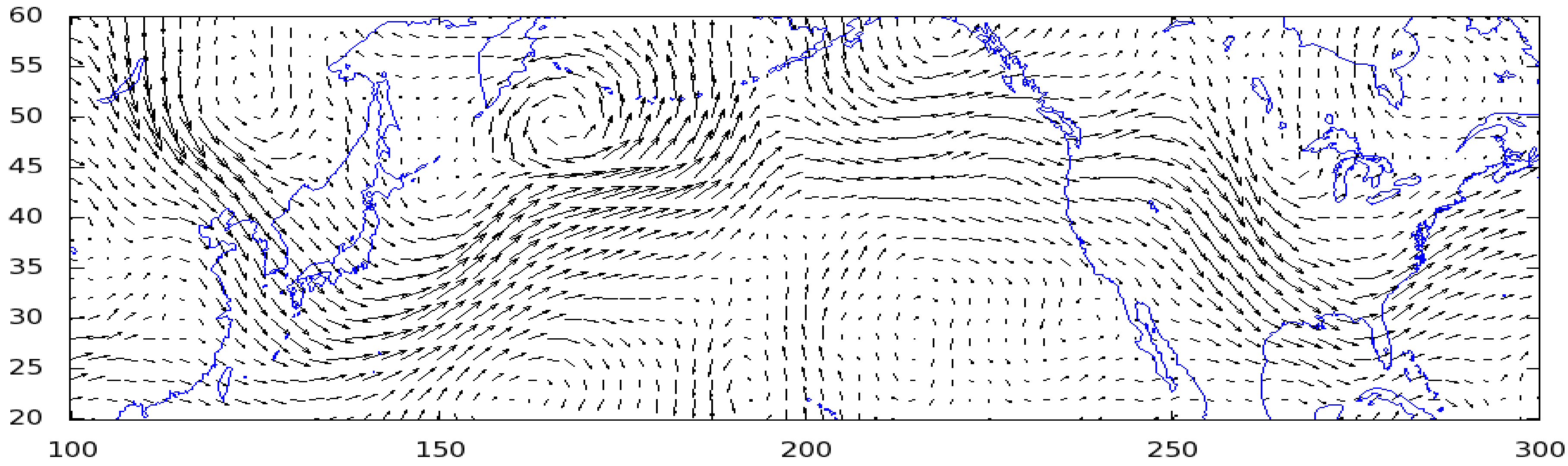
$c = const$ along the trajectory $\dfrac{d\boldsymbol{r}}{dt} = \boldsymbol{u}(\boldsymbol{r}, t)$



Velocity Field



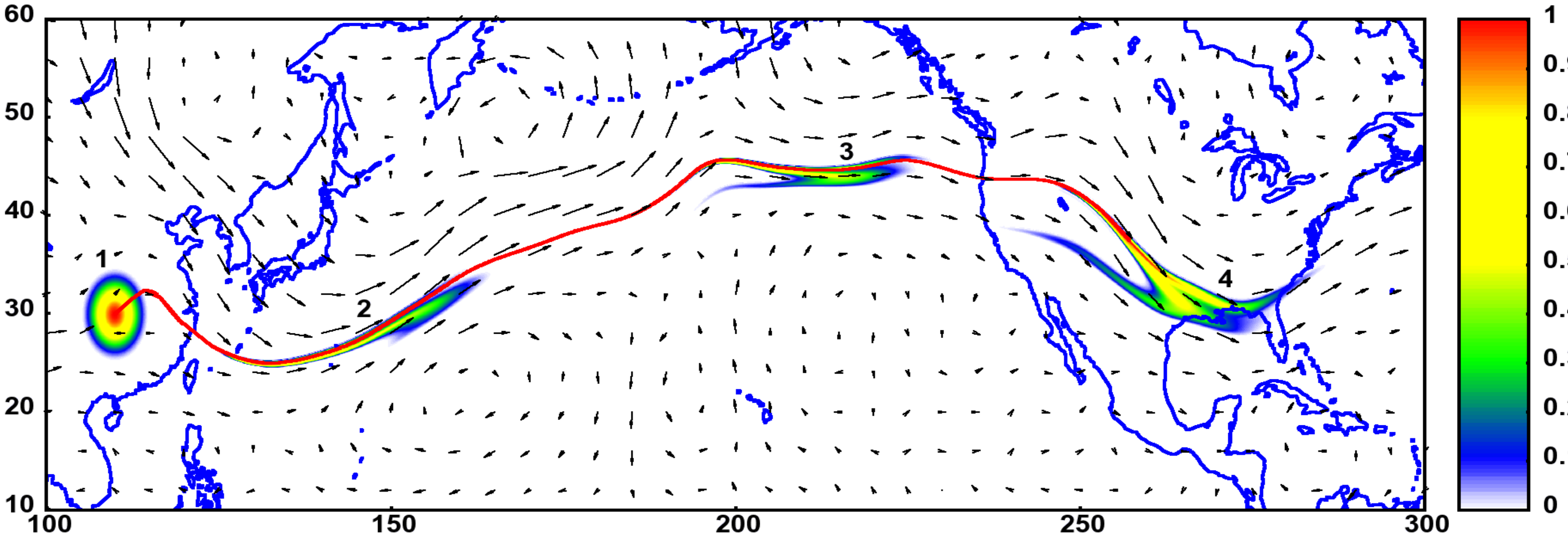
Global domain



Pacific region

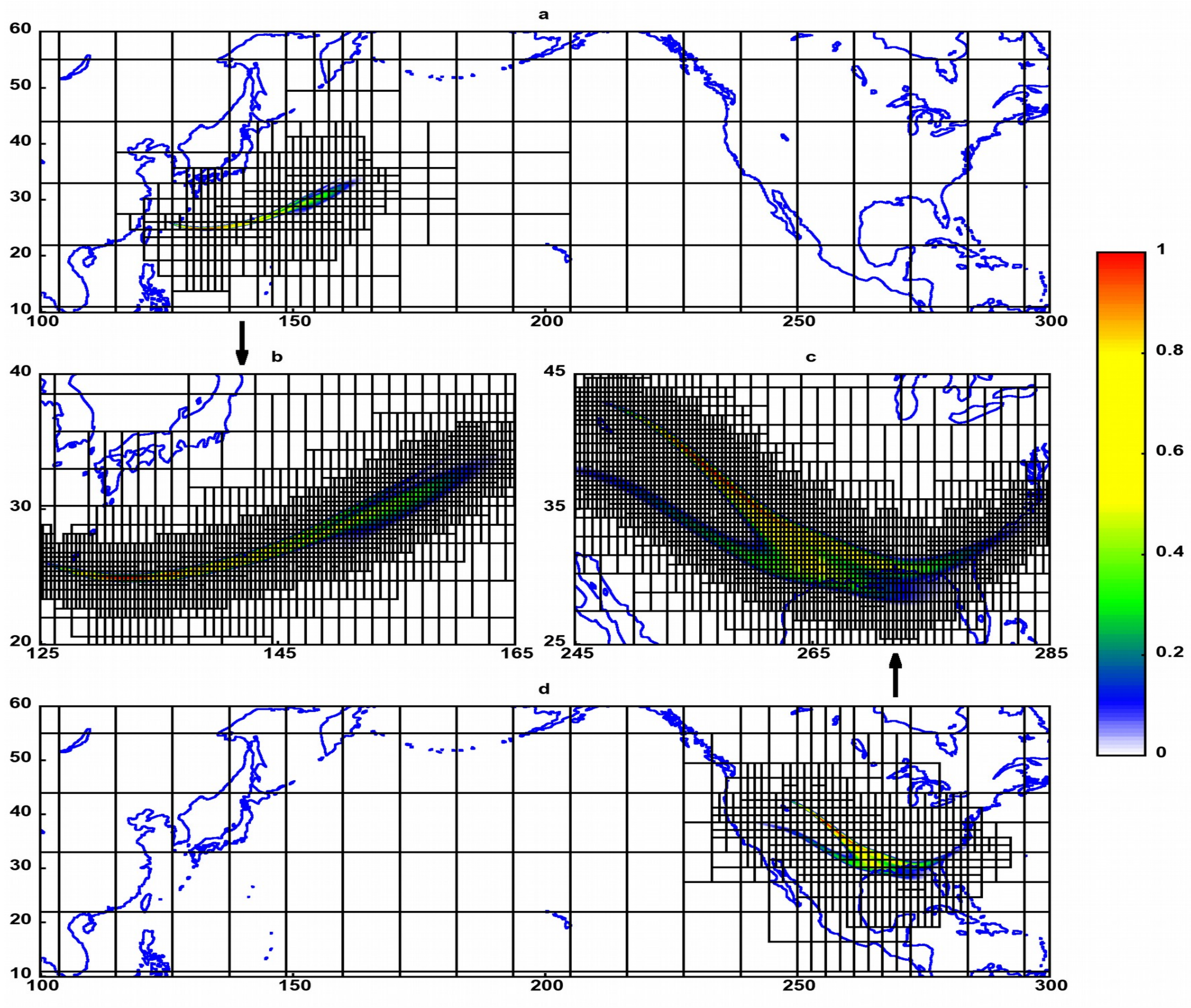
2D Atmospheric Flow

Stationary Velocity Field



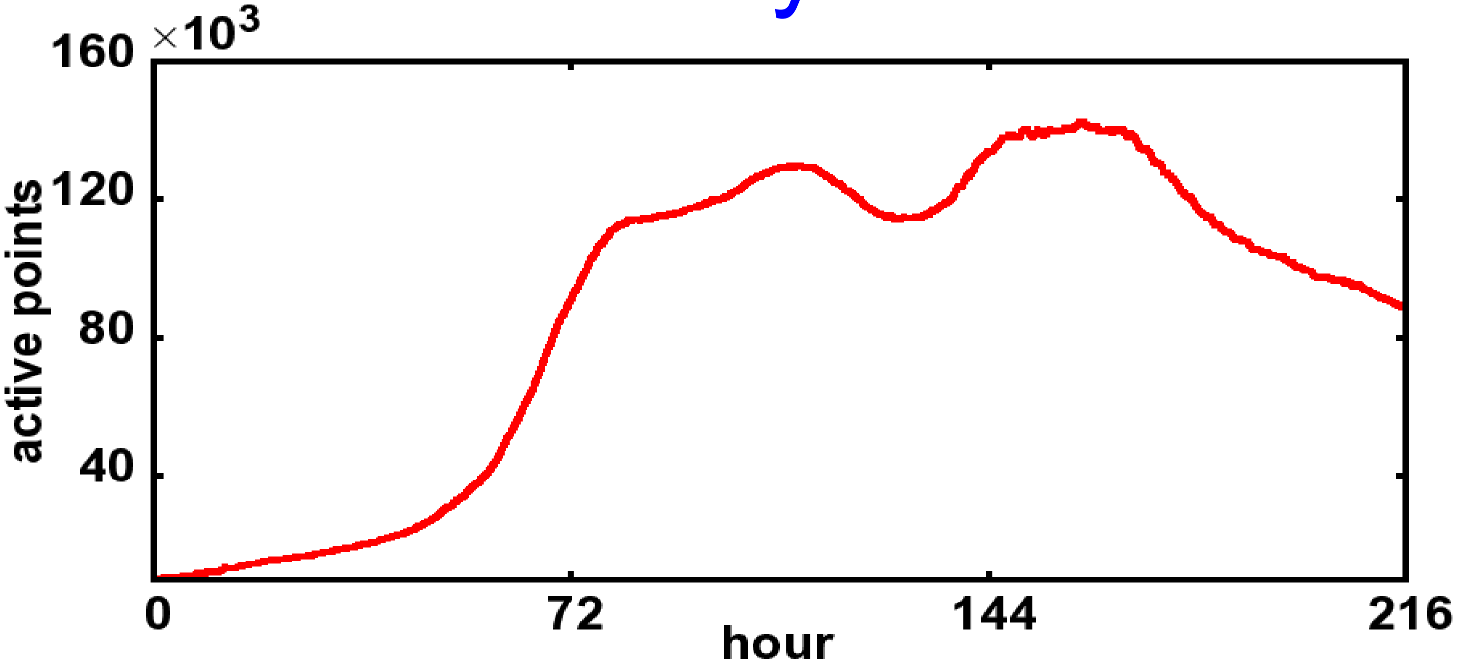
Evolution of a plume at different time moments

2D Non-Uniform Multilevel Grid

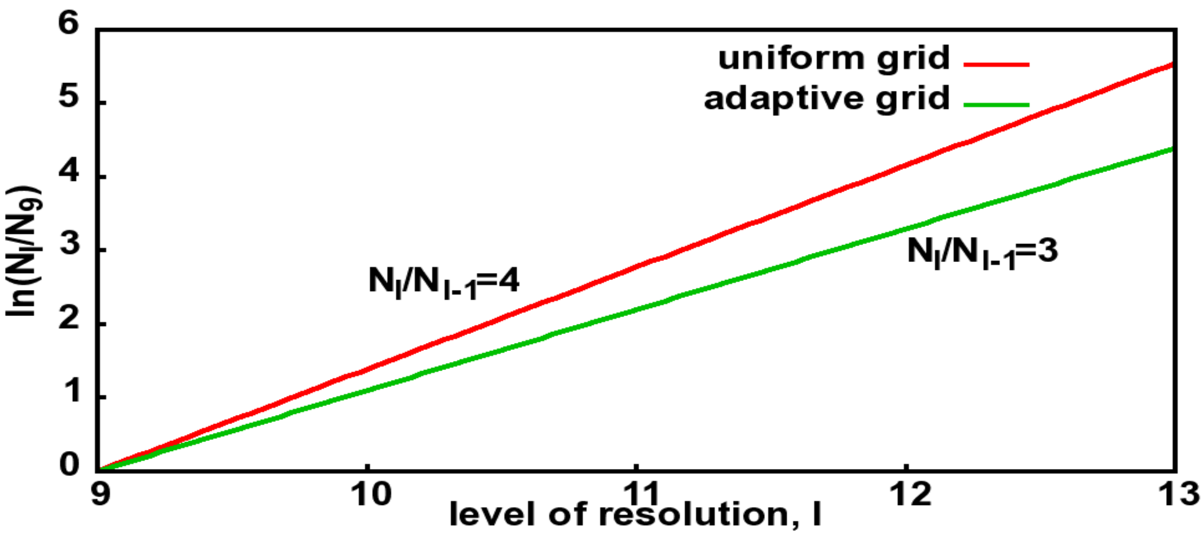


Evolution of 2D Transpacific plume

Grid Dynamics



Number of active points for the 2D adaptive grid



Increase of grid size N with resolution l

Compression ratio ~1000 for 2D flows!!!!

Thank you