

# Assessment of turbulent wake for two wind turbines using multi-fractal analysis

A.S. Kryuchkova, S.V. Strijhak,  
J. Tellez-Alvarez, Jose M. Redondo

ISPRAS (Russia, Moscow), UPC (Spain, Barcelona)

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- Introduction to Wind Energy
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# Wind Energy in the world community

**Countries:** Denmark, Germany, Netherlands, Spain, USA,  
China, India

**RUSSIA is at the beginning stage (Fortum/Rosnano and Rosatom)**

**Main Scientific groups:**

*Technical University Denmark* – Prof. Sørensen Jens Nørkær

*Johns Hopkins University* - Prof. Charles Meneveau

*École Polytechnique Fédérale de Lausanne (EPFL)* –

Prof. Fernando Porté-Agel

*TU Delft, NTNU (Norway), KTH, University of Exeter,*

*Portland State University (USA), CENER (Spain)*

**Manufactures:**

Vestas (Denmark), Enercon (Germany),

Siemens Wind (Germany), Gamesa (Spain),

NREL (USA), GE Energy (USA),

Sinovel (China), Suzlon Energy (India)

**Research projects:** EU ESWIRP, EU AVATAR, MesoWake

**Commercial codes:** windPro



**General Topics in Wind Energy:**

1. Materials and structures
2. Wind and turbulence
3. Aerodynamics
4. Control and system identification
5. Electricity conversion
6. Reliability and uncertainty modelling
7. Design methods
8. Hydrodynamics, soil characteristics and floating turbines
9. Offshore environmental aspects
10. Wind energy in the electric power system
11. Societal and economic aspects

# Wind and turbulence, aerodynamics

**Scientific challenge 2-1:** *To work out key parameters across scales that enable a rigorous approach to characterize wind conditions (**wind gusts are not taken into account**).*

**Scientific challenge 2-2:** *To understand small-scale turbulence for instationary atmospheric and complex orographic conditions. (**the turbulence intermittency problem**)*

**Scientific challenge 2-3:** *To be able to set up the correct reduced models based on the right scales to comprehend all important features of wind fields in space and time.*

**Table 1.** Scale requirements in wind turbine aerodynamics

	Length scale (m)	Velocity scale ( $\text{m s}^{-1}$ )	Timescale (s)
Aerofoil boundary layer	0.001	100	0.00001
Aerofoil	1	100	0.01
Rotor	100	10	10
Cluster	1000	10	100
Wind farm	10 000	10	1000
Cluster of wind farms	100 000	10	10 000

**Scientific challenge 3-1:** *To work out rigorous methods in order to be able to exploit the growing computational power and experimental advances for clarifying challenging problems in aerodynamics (**different lengths and timescales are involved**).*

**Scientific challenge 3-2:** *To achieve an understanding of 3-D flow pattern and their dynamics on rotor blades including emerging turbulence.*

**Scientific challenge 3-3:** *To achieve a fundamental understanding of the aerodynamics of wakes and interacting wakes behind wind turbines (**flow control devices, airfoil aeroacoustics**).*

## Blind tests experiments\*



\*Krogstad P.-Å., Eriksen P.E.; "Blind test" calculations of the performance and wake development for a model wind turbine. Renewable Energy. vol. 50., 2013

\* Pierella F., Krogstad P.-Å., Sætran L.R.; Blind Test 2 calculations for two in-line model wind turbines where the downstream turbine operates at various rotational speeds. Renewable Energy. vol. 70., 2014

# 2015 “Blind test4” Workshop

Different position of wind turbines:

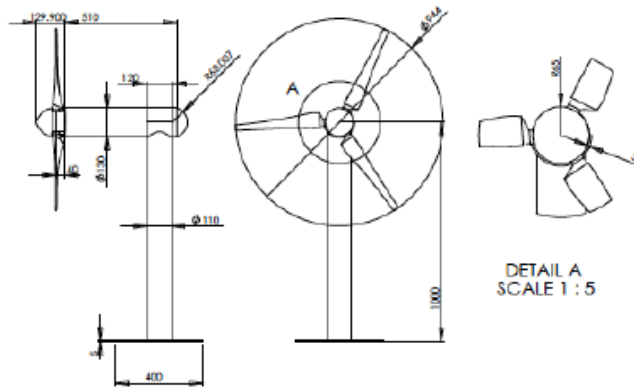
- $x/D = 2.77$   $x/D = 5.18$   $x/D = 9.00$
- **Low turbulence uniform inflow:**  $Tu=0.23\%$
- **High turbulence uniform inflow:**  $Tu= 10.0\%$
- **High turbulence shear inflow:** (A turbulence grid with increasing vertical distance between the horizontal bars is installed at the inlet)  $Tu = 10.1\%$
- The blades were machined in aluminum and have a NREL S826 airfoil section from root to tip
- The tower of turbine  $T1$  is a cylinder with a constant diameter of  $DTow,T1= 0.11m$ , while for  $T2$  the rotor sits on top of a stepped tower consisting of 4 cylinders of different diameters.
- $T2$  is the same turbine that was used in BT1.
- The nacelle of turbine  $T1$  is a circular cylinder of  $DNac,T1 = 0.130m$  diameter.
- The nacelle of  $T2$  is also circular but with a diameter of  $DNac,T1 = 0.130m$ .
- The rotor diameter of  $T1$  is  $DT1 = 0.944m$ , while  $DT2 = 0.894m$ .
- The individual blades have a total length of  $lBlade = 0.413m$  and are directly mounted on the hubs with the diameters  $Dhub,T1= 0.118m$  and  $Dhub,T2= 0.068m$ .



Fig. Model wind turbines in the wind tunnel ( $x/D=9.00$ ) in NTNU, Trondheim, Norway

The model turbines were tested in a closed-return wind tunnel. It has a test section which is 2.71m wide and 11.15m long.

# Data from experiment



*Figure 2 (a): Tower and nacelle dimensions of the upstream turbine  $T_1$*

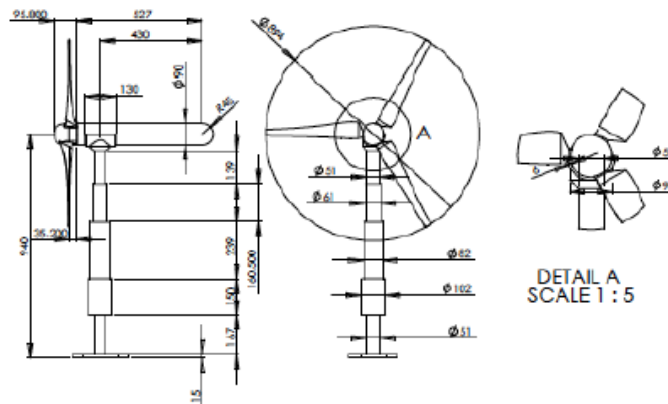


Figure 2 (b): Tower and nacelle dimensions of the downstream turbine  $T_2$

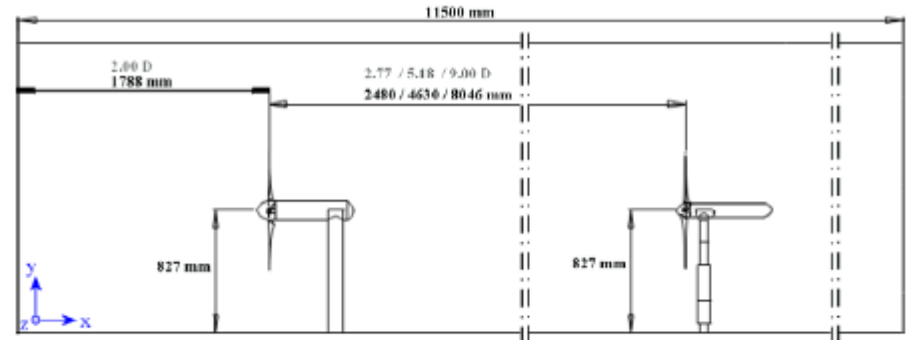
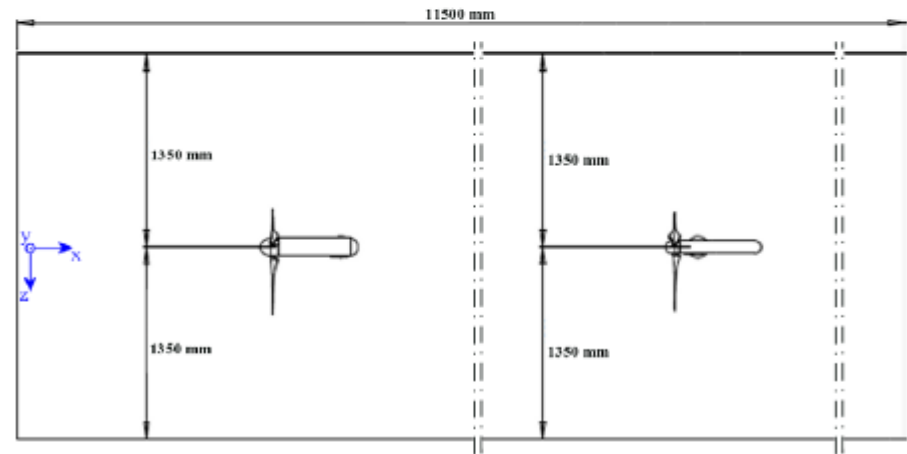


Figure 3: Turbine positions in the wind tunnel and reference coordinate system

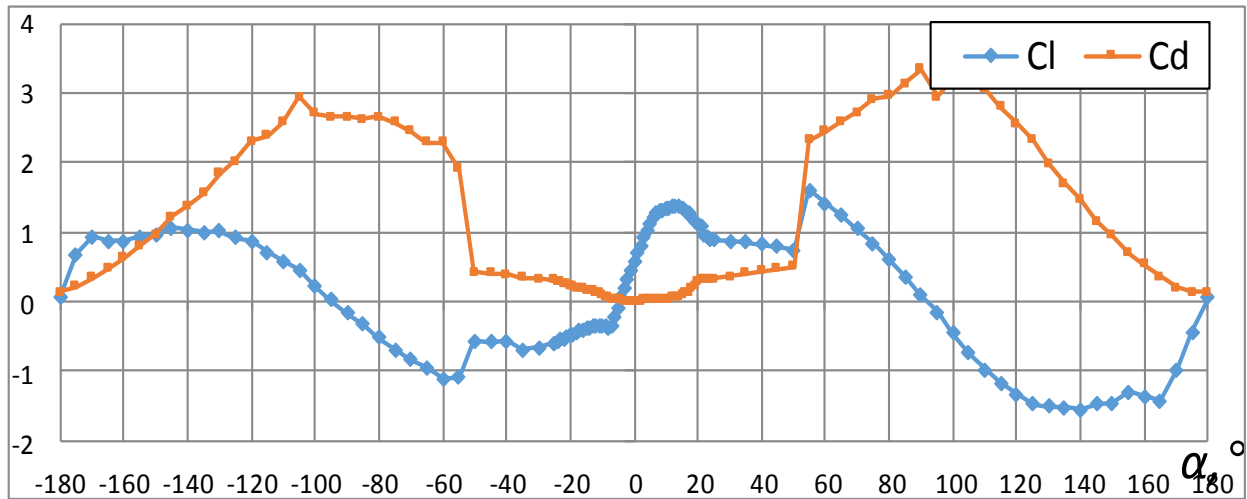
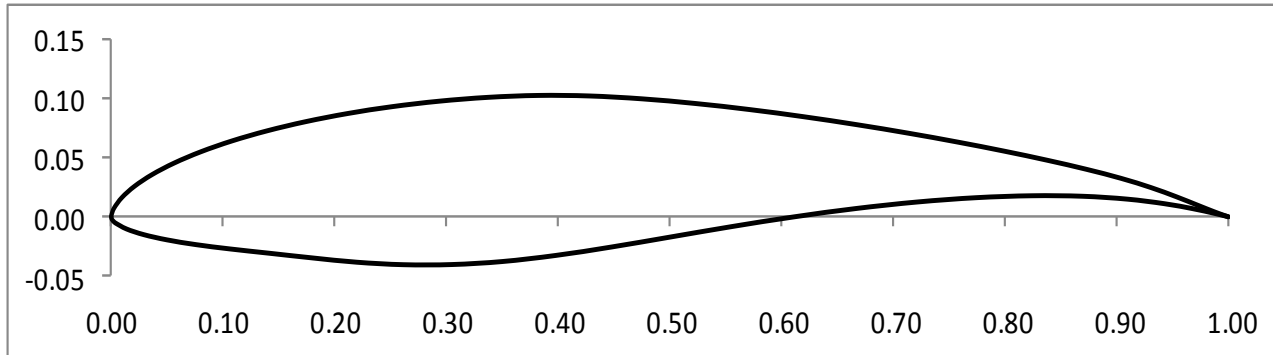


*Figure 4: Wind tunnel test section from above and reference coordinate system*

Fabio Pierella, Per-Åge Krogstad, Lars Sætran. Blind Test 2 calculations for two in-line model wind turbines where the downstream turbine operates at various rotational speeds. *Renewable Energy* 70 (2014) 62-77

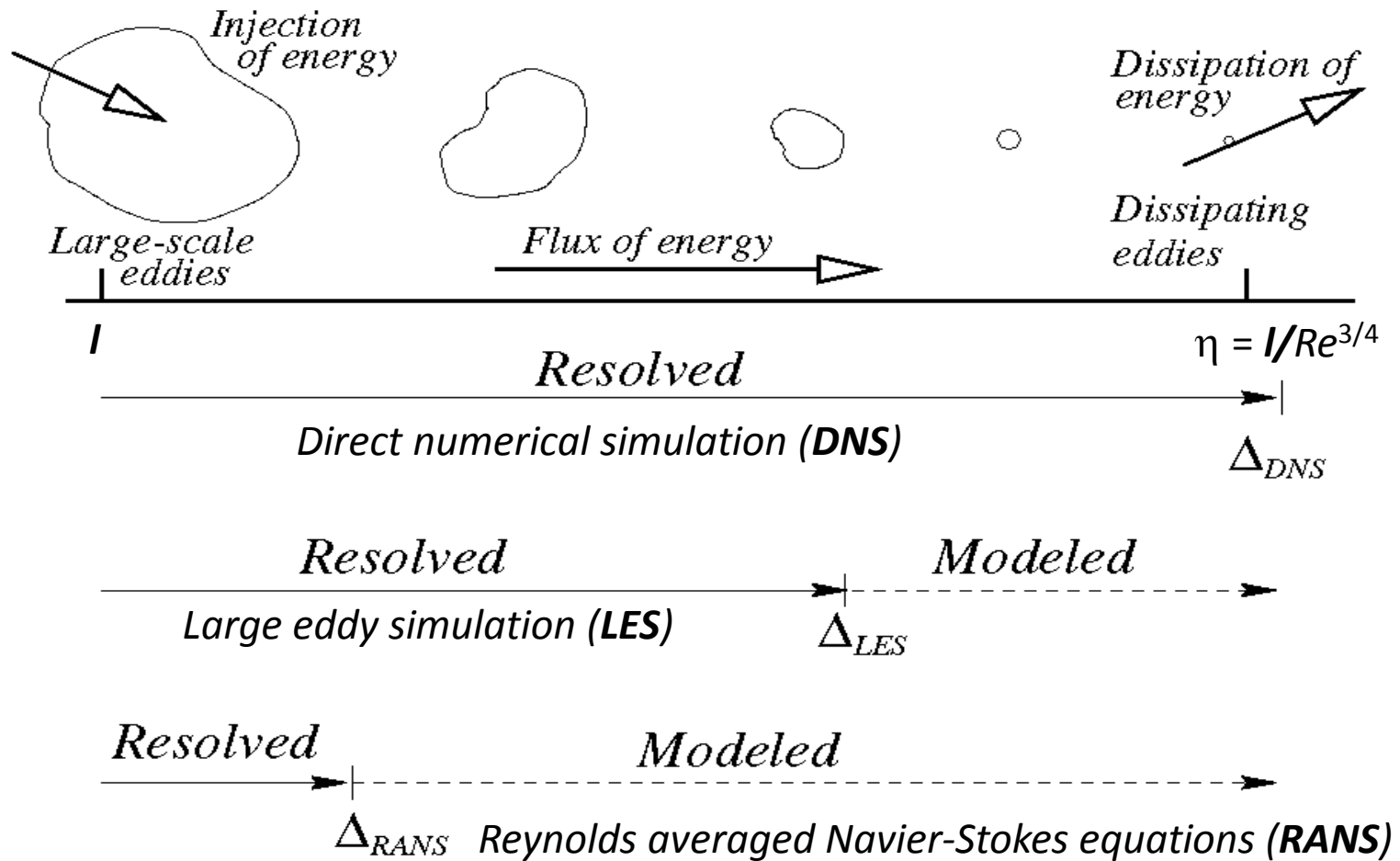


# Airfoil NREL S826





# Prediction Methods



## Filtered transport equations in LES

- The filtered continuity and momentum equations use filtered variables:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho \tilde{u}_j}{\partial x_j} = 0$$

and:

$$\frac{\partial \rho \tilde{u}_i}{\partial t} + \frac{\partial \rho \tilde{u}_i \tilde{u}_j}{\partial x_j} = - \frac{\partial \tilde{p}}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} + \frac{\partial \sigma_{ij}}{\partial x_j}$$

$\tau_{ij}$  is the filtered stress tensor.

$\sigma_{ij}$  are the subgrid-scale Reynolds stresses.

# Filtered variables

- A variable,  $\phi(\mathbf{x}')$ , is filtered using a filter function,  $G$ .

$$\tilde{\phi}(\mathbf{x}) = \int_D \phi(\mathbf{x}') G(\mathbf{x}, \mathbf{x}') d\mathbf{x}'$$

- $G$  is a function of the cell volume.

$$G(\mathbf{x}, \mathbf{x}') = \begin{cases} 1/V & \text{for } \mathbf{x}' \in V \\ 0 & \text{otherwise} \end{cases}$$

Thus:

$$\tilde{\phi}(\mathbf{x}) = \frac{1}{V} \int_V \phi(\mathbf{x}') d\mathbf{x}', \quad \mathbf{x}' \in V$$

# Software SOWFA\* for numerical simulation of flow parameters

- Different solvers in OpenFoam
- Simulation of flow parameters in Atmospheric Boundary Layer - **ABLSolver**
- Wind turbine simulation - **pisoTurbineFoam**
- Wind plant simulation - **windPlantSolver**
- Wind plant simulation with FSI (FAST solver) – **windPlantSolverFast**
- Large Eddy Simulation (LES) model
- Different SGS models for LES
- Special utilities for boundary conditions
- Scales from  $10^{-3}$  m till  $10^5$  m
- $Re \sim 10^7$

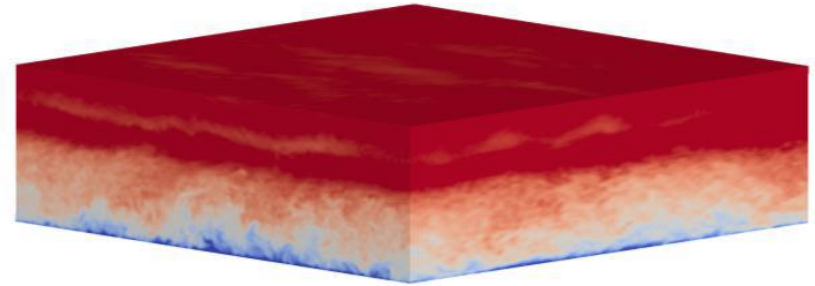


Fig. Velocity magnitude

Domain: 3 km x 3 km x 1 km  $V=8$  m/c at  $H=90$  m.  
Solver ABLSolver.

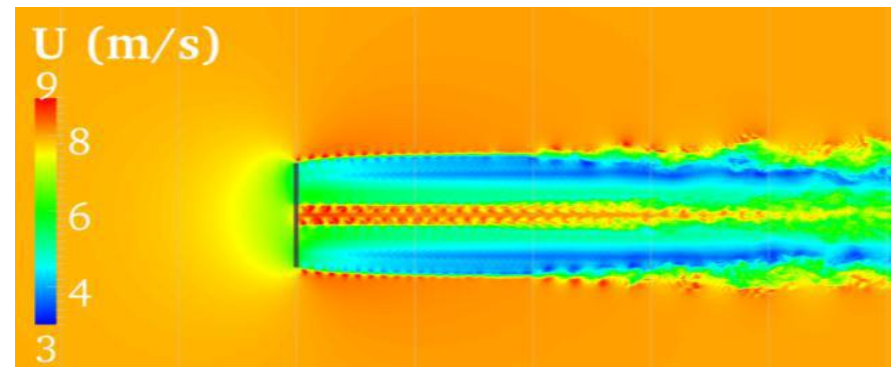


Fig. Velocity field. Solver pisturbineFoam

\* SOWFA, The Simulator for Wind Farm Applications, created by The National Renewable Energy, distributed online at <https://wind.nrel.gov/SOWFA>, 2016.

# Mathematical model in SOWFA: Finite Volume Method and Large Eddy Simulations

$$\frac{\partial \bar{u}_j}{\partial x_j} = 0 \quad - \text{mass conservation equation}$$

$$\bar{u}_j = u_j - u'_j \quad - \text{velocity after procedure of filtration}$$

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial}{\partial x_j} (\bar{u}_j \bar{u}_i) = -2\varepsilon_{ijk} \Omega_j \bar{u}_k - \frac{\partial \tilde{p}}{\partial x_i} - \frac{\partial}{\partial x_j} (R_{ij}^D) + \left( \frac{\rho_b}{\rho_0} - 1 \right) g_i - \left\langle \frac{\partial p}{\partial x_i} \right\rangle + f_i \quad - \text{momentum equation}$$

Where  $\varepsilon_{ijk}$  - the alternating tensor,  
 $\Omega_j$  - Rotation Rate Vector for Earth,  
 $\tilde{p}$  - Modified pressure variable,  
 $R_{ij}^D$  - Fluid stress tensor.

$$\frac{\partial \bar{\theta}}{\partial t} + \frac{\partial u_j \bar{\theta}}{\partial x_j} = - \frac{\partial \tau_{\theta_i}}{\partial x_j} \quad - \text{a potential temperature transport equation}$$

Where  $\bar{\theta}_j$  - the resolved-scale potential temperature,  $\tau_j$  - is the SGS temperature flux

# Initial Conditions for Precursor Flow

- **Velocity**

- Given a logarithmic base profile
- Non-random, divergence-free perturbations added near surface to cause turbulence to quickly happen (similar to method used by DeVillier's in channel flow).

- **Temperature**

- Constant temperature (300K) up to some height, then temperature increases
- This creates a capping inversion that caps the boundary layer and slows boundary layer vertical growth

- **Pressure variable**

- Initialized to zero

- **Initial conditions set using “setABLFIELDS” utility, but could use something like “funkySetFields”**

De Villiers, E., “The Potential of Large Eddy Simulation for the Modeling of Wall Bounded Flows”, PhD Thesis, Imperial College, London, 2006.

# Numerical Scheme

## Finite-volume formulation

- Linear interpolate of cell-center values to cell faces when needed
- Equivalent to second-order central differencing
- Rhie-Chow\* like flux interpolation is used to avoid pressure-velocity decoupling

## Linear System Solvers

### Velocity and Temperature –

- Biconjugate Gradient
- Diagonal incomplete LU matrix preconditioner

### Pressure

- Preconditioned Conjugate Gradient
- Diagonal incomplete Cholesky matrix preconditioner

### **OR**

- Geometric agglomerated algebraic multigrid solver
- Diagonal incomplete Cholesky smoother

\* C. M. Rhie and W. L. Chow. Numerical Study of the Turbulent Flow Past an Airfoil with Trailing Edge Separation. *AIAA Journal*, Vol. 21, No. 11, 1983, pp. 1552–1532.



# Solver outputs

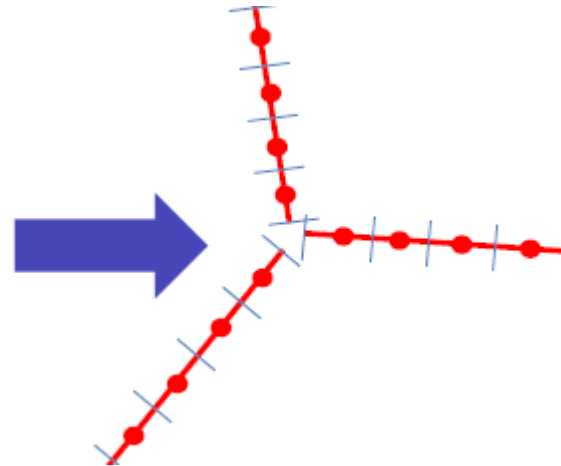
“averaging” file structure

Cell-center quantities	Description
T_mean	$\langle \bar{\theta} \rangle$
U_mean, V_mean, W_mean	$\langle \bar{u} \rangle \langle \bar{v} \rangle \langle \bar{w} \rangle$
uu_mean, vv_mean, ww_mean	$\langle u'u' \rangle \langle v'v' \rangle \langle w'w' \rangle$
uv_mean, uw_mean, vw_mean	$\langle u'v' \rangle \langle u'w' \rangle \langle v'w' \rangle$
wuu_mean, wvv_mean, www_mean	$\langle w'u'u' \rangle \langle w'v'v' \rangle \langle w'w'w' \rangle$
wuv_mean, wuw_mean, wwv_mean	$\langle w'u'v' \rangle \langle w'u'w' \rangle \langle w'v'w' \rangle$
Tu_mean, Tv_mean, Tw_mean	$\langle \theta'u' \rangle \langle \theta'v' \rangle \langle \theta'w' \rangle$

Cell-face quantities	Description
R11_mean, R22_mean, R33_mean	$\langle R_{11}^D \rangle \langle R_{22}^D \rangle \langle R_{33}^D \rangle$
R12_mean, R13_mean, R23_mean	$\langle R_{12}^D \rangle \langle R_{13}^D \rangle \langle R_{23}^D \rangle$
q1_mean, q2_mean, q3_mean	$\langle R_{\theta 1} \rangle \langle R_{\theta 2} \rangle \langle R_{\theta 3} \rangle$
phiM	$\phi_m$ Non-dimensional velocity shear

Global quantities	Description
ReLES	$Re_{LES}$ LES Reynolds number <sup>1</sup>
scriptR	$\mathfrak{R}$ Near surface ratio of resolved to subgrid scale stress <sup>1</sup>
uStar	$u_*$ Friction velocity
zi	$z_i$ Boundary layer depth

# Actuator Line Model for wind turbine



$$f_i^{turbine}(r) = \frac{F_i^{actuator}}{\varepsilon^3 \pi^{3/2}} \exp \left[ - \left( \frac{r}{\varepsilon} \right)^2 \right]$$

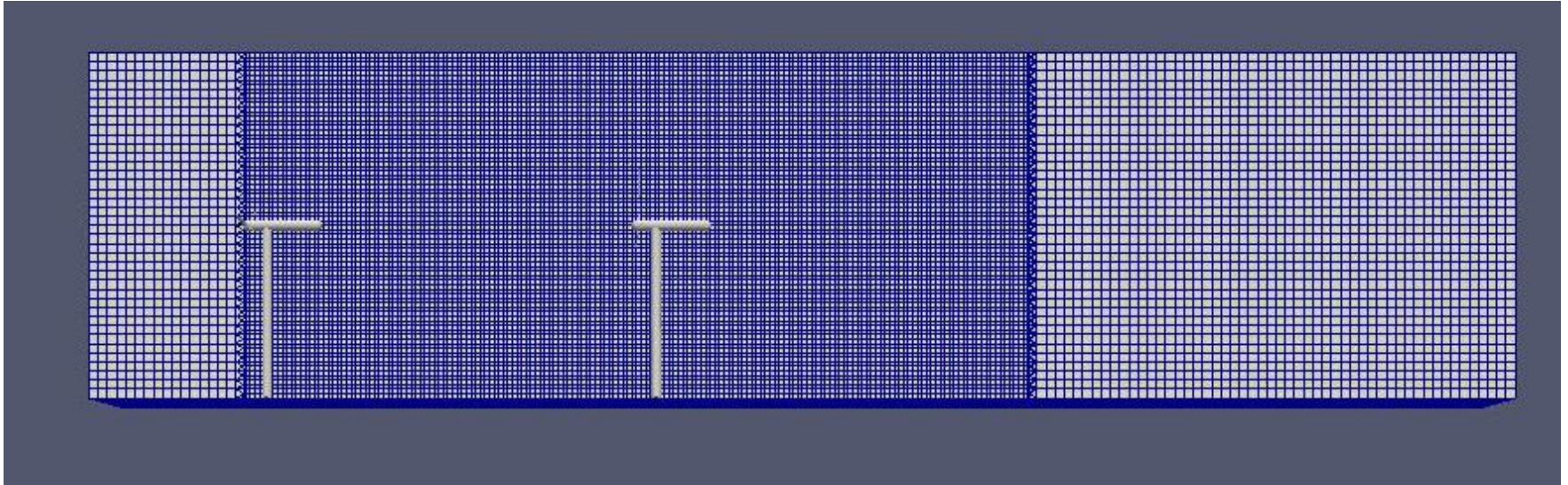
Total Aerodynamic Force

Aerodynamics coefficients  $C_x(\alpha)$   $C_y(\alpha)$

Angle of Attack from -180 till 180. Simple bodies for wind turbine:

"Cylinder1", "Cylinder2", airfoil profiles "DU40\_A17", "DU35\_A17", "DU30\_A17", "DU25\_A17", "DU21\_A17", "NACA64\_A17"

# The domain for simulation



$$U_{\text{REF}} = 11.5 \text{ m/c.}$$

$$\lambda_{T1} = 6 \text{ и } \lambda_{T2} = 4.5$$

$$Re = 10^5$$

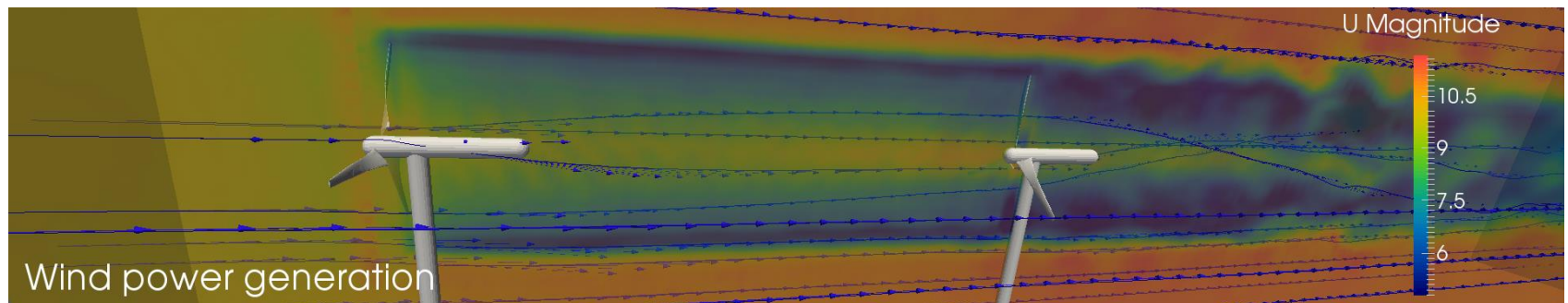
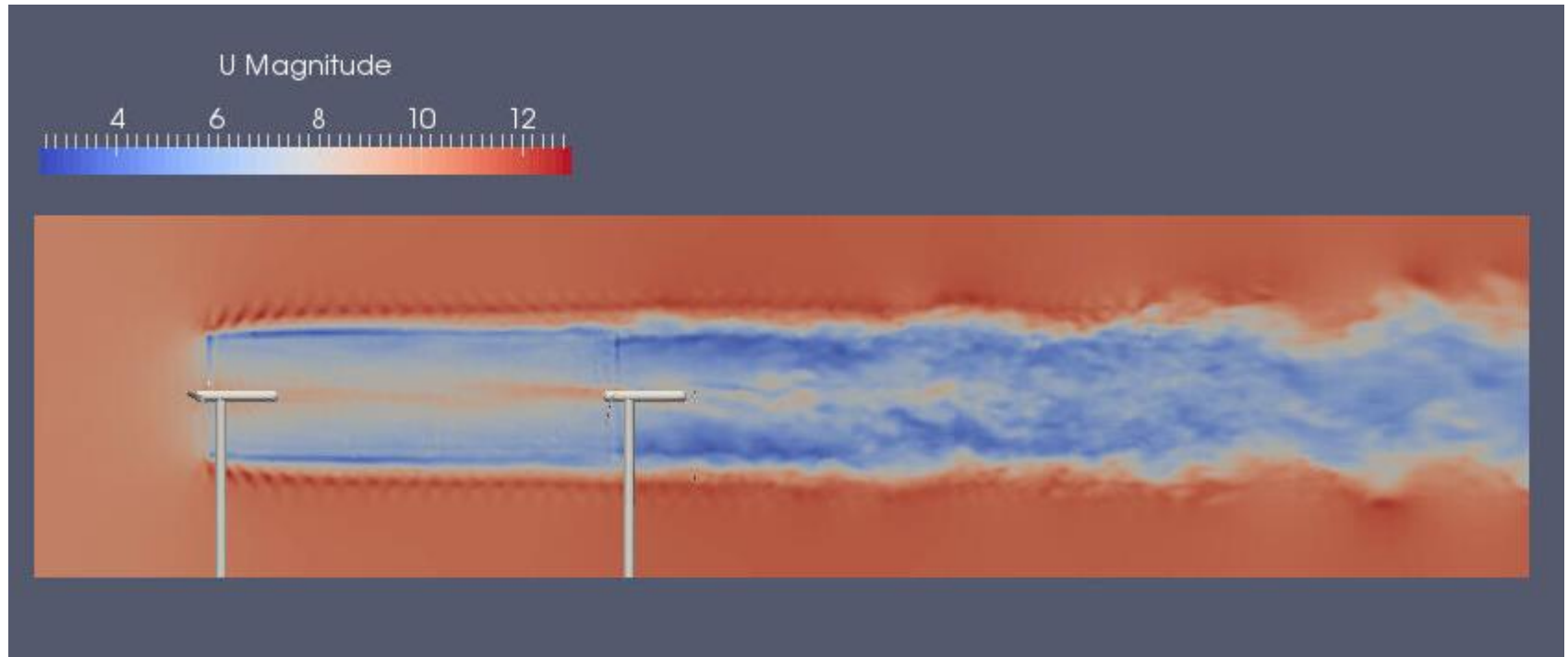
$\sim$

The numerical domain was defined from: (0 -1.355 0) to (11.15 1.355 3.6) meters

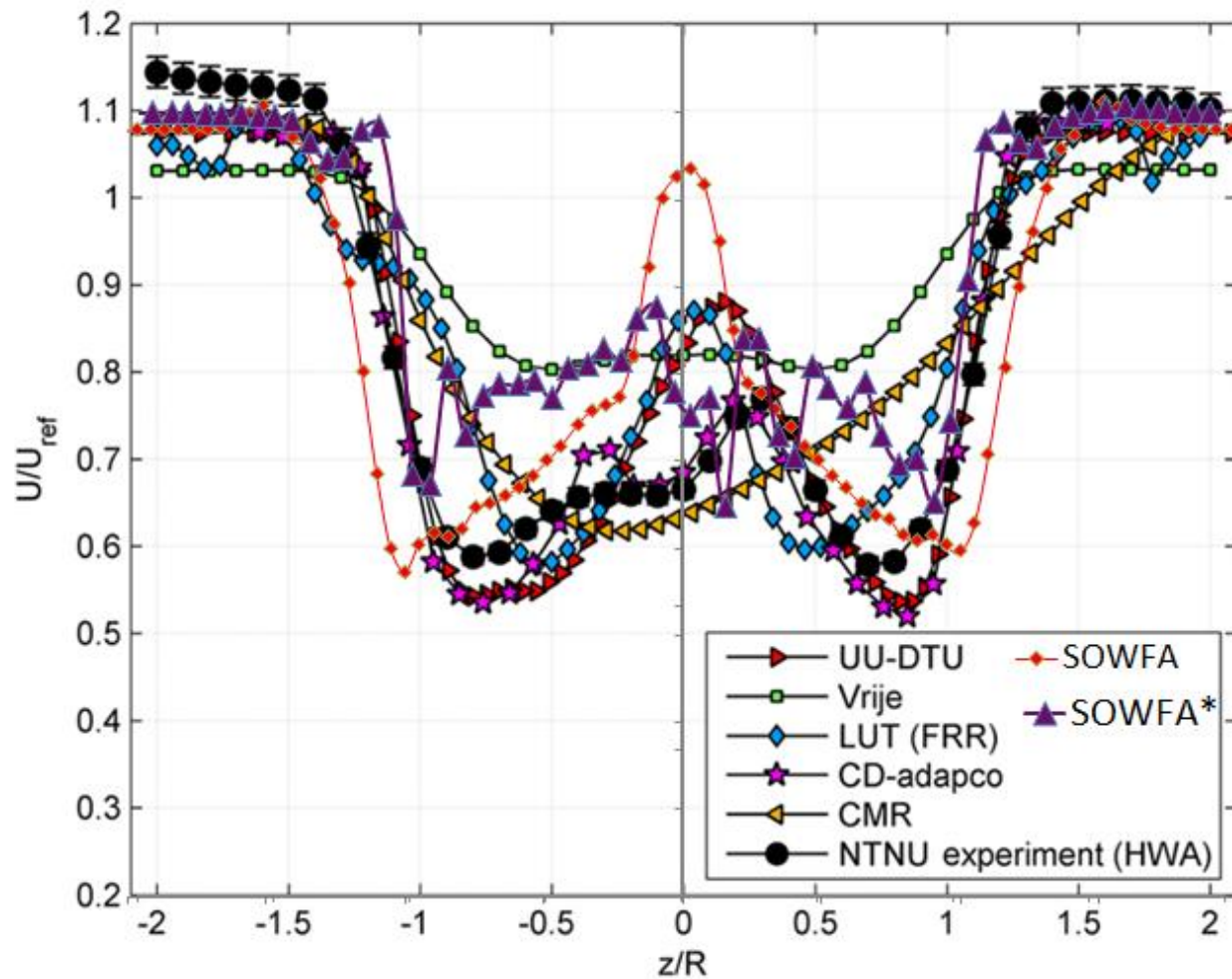
$$x/D = 2.77, 5.18, 9.0$$

Mesh: 2, 4, 8 mln. cells

# U magnitude at t=2 seconds

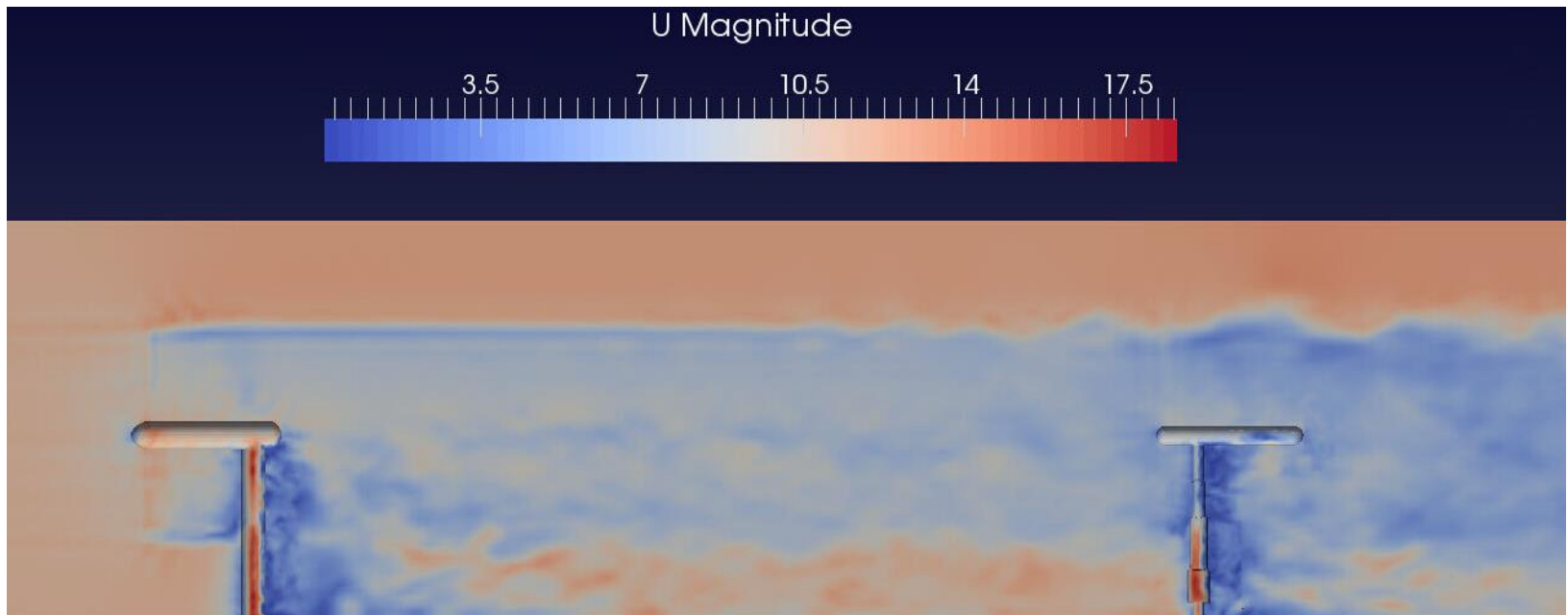


*$U/U_{ref}$  values at  $x/D=2.77$*



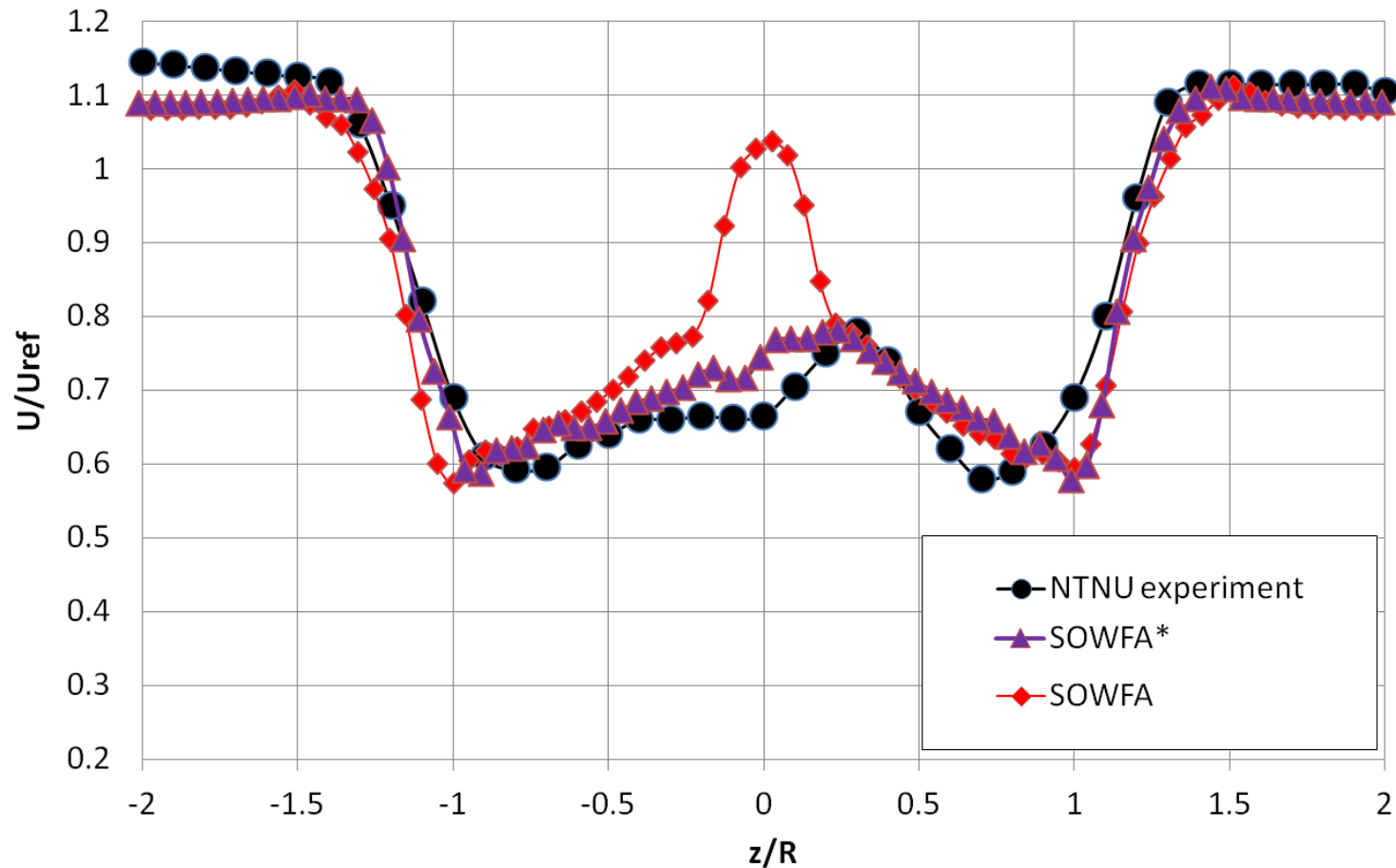


# U magnitude at t=2 seconds



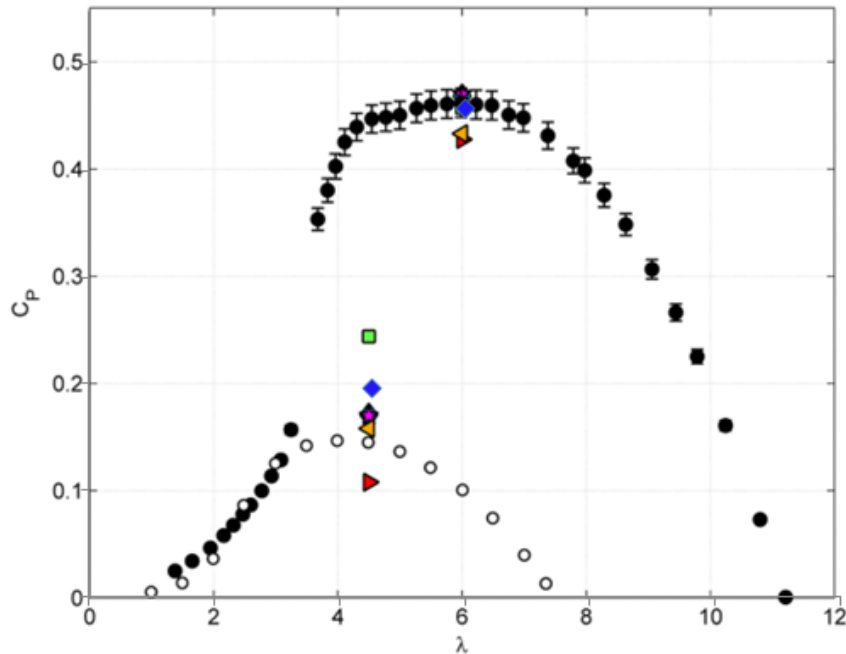
Tower and HUB of wind turbine are resolved  
The mesh was generated using snappyHexMesh

# *Comparison of horizontal field profile of normalized velocity*

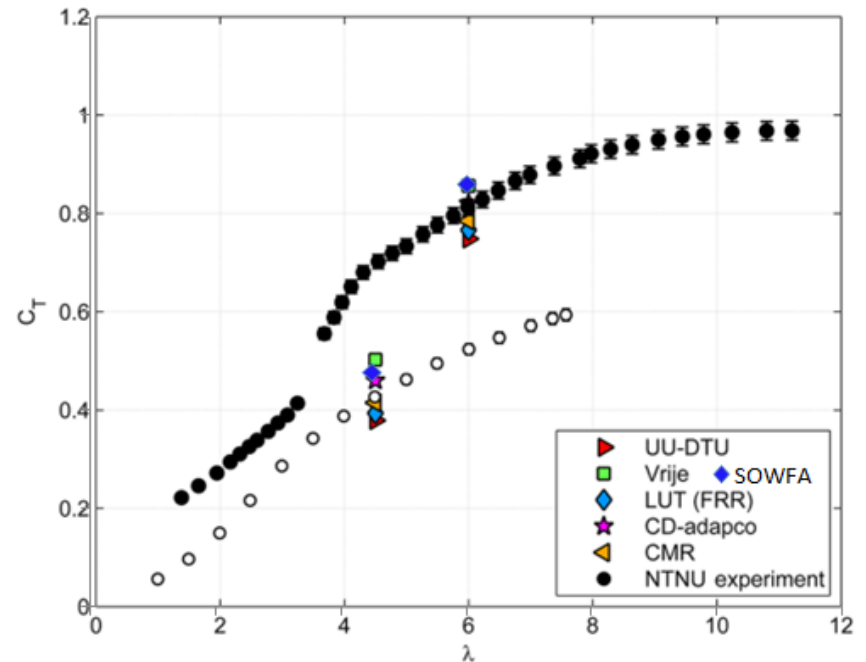




# Coefficients of power $C_p$ and axial thrust $C_T$ .



$$C_{P,T1/T2} = \frac{8 \cdot P_{T1/T2}}{\rho \pi D_{T1/T2}^2 U_{ref}^3}$$



$$C_{T,T1/T2} = \frac{8 \cdot F_{T1/T2}}{\rho \pi D_{T1/T2}^2 U_{ref}^2}$$

# FRACTAL ANALYSIS

✿ Fractal studies provide a natural method for analysing turbulent fields like ABL, wakes, plumes and their turbulent cascade processes.

✿ If there is a subrange where production and dissipation are at equilibrium, it is possible a functional relation between the exponent  $\beta$  of the spectral density function and the fractal dimension  $D$  of the scalar field represented in the images:

$$\beta = 2E_U + 1 - 2D \Rightarrow D = E_U + \frac{1 - \beta}{2}, \quad E_U : \text{Euclidian dimension}$$

✿ The last aim is to investigate the intermittency of the mixing ABL (measuring the maximum fractal dimension and using results of another researchers relating to the sixth and third order structure function scaling exponents).

The structure function, the intermittency exponent,  
the Hausdroff dimension

$$B(R)^P = \left\langle \left( (u(x+R) - u(x))^P \right) \right\rangle = \left\langle \delta_R(u)^P \right\rangle \quad \text{The structure function}$$

$$B(R)^P \sim \langle \varepsilon \rangle^{P/3} R^{P/3}$$

$$B(R)^P \sim \langle \varepsilon_R \rangle^{P/3} R^{P/3} \sim R^{\xi_p}$$

$$\xi_p = \frac{p}{3} + \frac{1}{18} \mu p (3-p)$$

the scaling exponent

$$\mu = 2 - \xi_6$$

$\mu$  is the intermittency exponent

$$\xi_p = \frac{p}{3} + \frac{1}{3} (3-H)(3-p)$$

$H$  is the Hausdroff dimension

# Calculation of Fractal Dimensions

The Moment method uses mainly three functions:  $\tau(q)$  - mass exponent function,  $\alpha$  - coarse Hölder exponent, and  $f(\alpha)$  - multifractal spectrum

$$n(\delta) \propto \delta^{-D_0}$$

number  $n$  of features of a certain size  $\delta$

$$D_0 = \lim_{\delta \rightarrow 0} \frac{\log n(\delta)}{\log \frac{1}{\delta}}$$

the fractal dimension  $D_0$

$$\chi(q, \delta) = \sum_{i=1}^{n(\delta)} \mu_i^q(\delta) = \sum_{i=1}^{n(\delta)} m_i^q$$

the partition function  $\chi(q, \delta)$

$$\langle \tau(q) \rangle = \lim_{\delta \rightarrow 0} \frac{\log \langle \chi(q, \delta) \rangle}{\log(\delta)} = \lim_{\delta \rightarrow 0} \frac{\log \langle \sum_{i=1}^{n(\delta)} m_i^q \rangle}{\log(\delta)}$$

the mass exponent function  $\tau(q)$

$$D_q = \lim_{\delta \rightarrow 0} \frac{1}{1-q} \frac{\log \sum_{i=1}^{n(\delta)} m_i^q}{\log \delta}$$

generalized dimensions  $D_q$ ,

$$\langle \alpha(q) \rangle = \frac{d \langle \tau(q) \rangle}{dq}$$

The singularity index ( $\alpha$ )

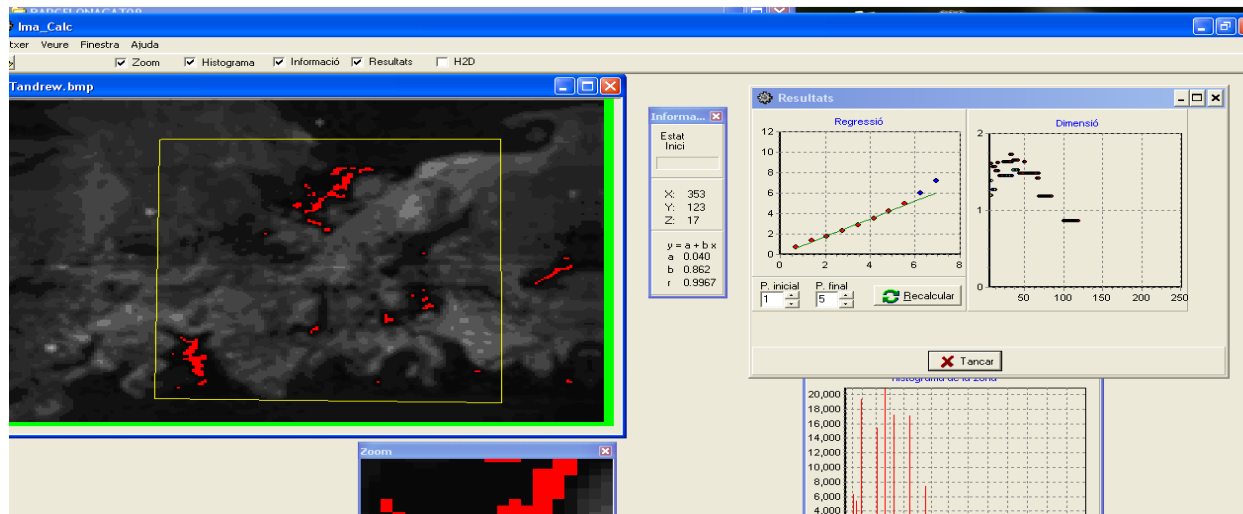
$$\langle f(\alpha) \rangle = q \langle \alpha(q) \rangle - \langle \tau(q) \rangle$$

$f(\alpha)$  is a scaling exponent of the cells with common  $\alpha$

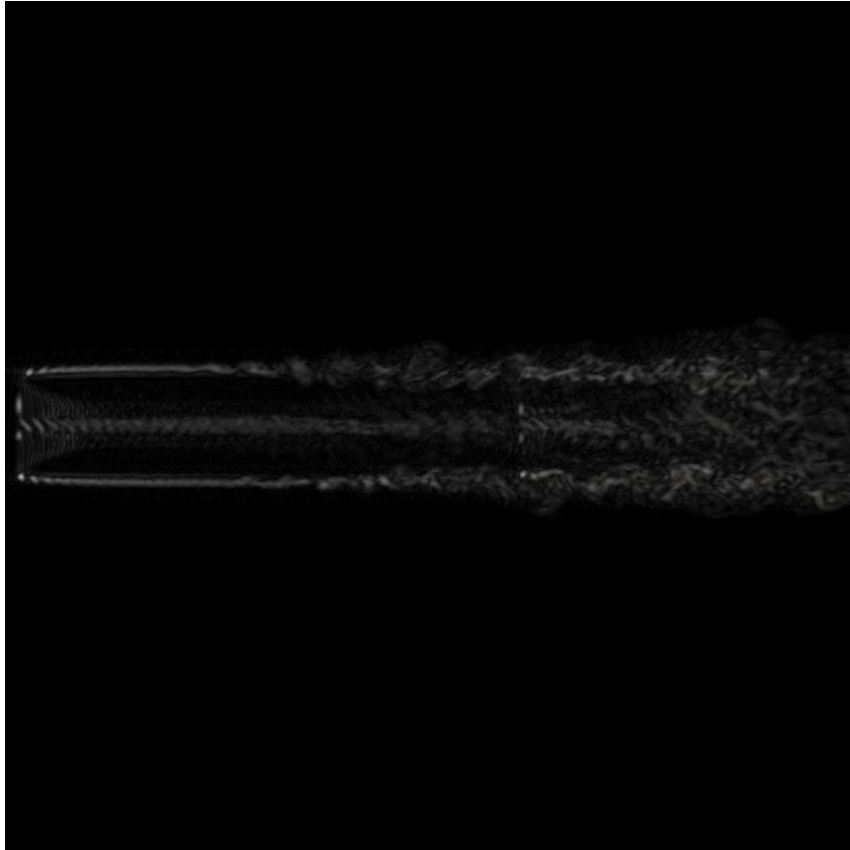
# ImaCal Program



the Box-Counting method built-in the ImaCal software (Grau, et al., 2003, Gade and Redondo 1999) is used to calculate the fractal dimension  $D_0$



# Vorticity at $t=2$ seconds

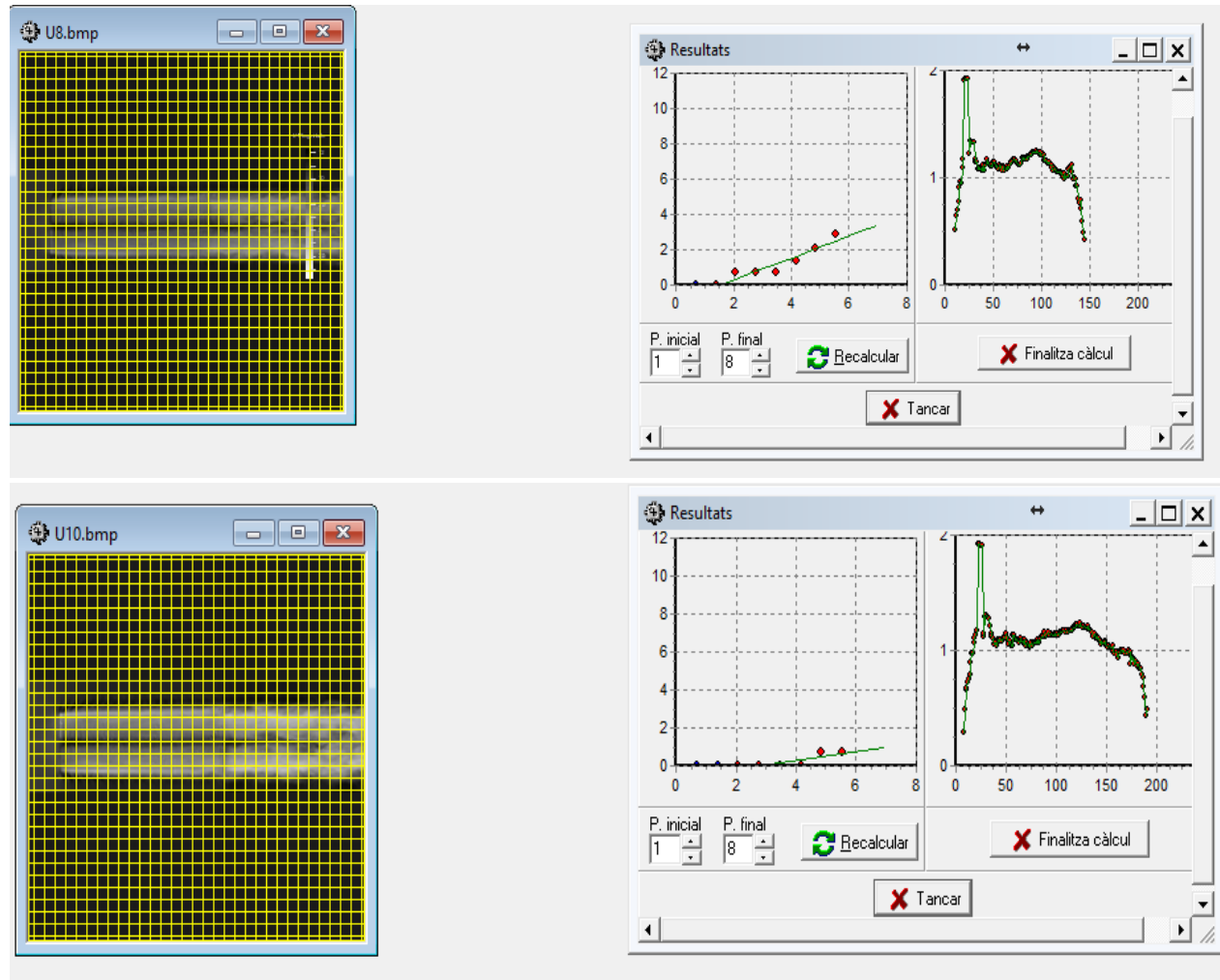


Mesh 2 mln. cells



Mesh 8 mln. cells

# Multifractal analysis of a wake for two wind turbines



The intermittency exponent changed for higher values of vorticity at an increasing rate, from 1.1 for low values up to the maximum of 2.1.



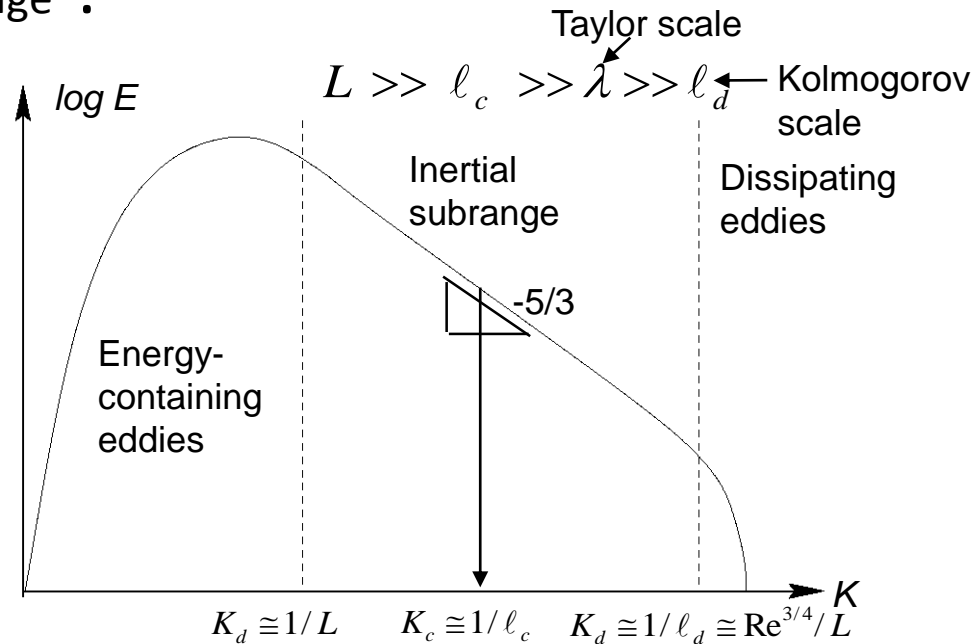
# Conclusions

- SOWFA library was installed on ISPRAS HPC cluster
- Several test cases (ABL GABLS, BLIND tests, NREL 5MW) were run with 12-96 cores
- Multifractal analysis of wind turbine was done with ImaCalc (BSC, Spain) program
- intermittency exponent is changed from 1.1 to 2.1
- Need more detailed analysis for turbulence statistical parameters ( $E(k)$ , Probability density function of velocity increments, statistics of velocity increments: skewness and flatness for hub height and tip regions)
- Better results for velocity in the core of the wake with resolved tower and hub
- Need new simulations on fine grids (10-80 mln. of cells) for 4, 9, 12, 24 wind turbines in wind plant
- Need HPC resources: 64-256 cores per case

This work is supported by RFBR grant № 17-07-01391.

# Large Eddy Simulation - what does it take?

- Requires 3-D transient modeling.
- Requires spatial and temporal resolution of scales in “inertial subrange”.



# Subgrid-scale (SGS) modeling

- SGS Reynolds stresses are modeled by:

$$\sigma_{ij}^s - \frac{1}{3} \delta_{ij} \sigma_{kk}^s = -2\mu_t S_{ij}$$

where  $\mu_t$  is the subgrid-scale eddy viscosity and  $S_{ij}$  is the rate of strain tensor.

- Common models are:

SGS (subgrid scale) stress tensor-eddy viscosity model

Smagorinsky SGS model

$$\mu_t = \bar{\rho} (c_s \Delta)^2 |S| \quad \Delta = (\Delta_x \Delta_y \Delta_z)^{\frac{1}{3}} \quad \mu_t = \rho L^2 \sqrt{2S_{ij}S_{ij}}$$

$$L = \min \left( \kappa d, C_s V^{\frac{1}{3}} \right)$$

One Eddy Equation model

$$\frac{\partial \bar{\rho} k}{\partial t} + \nabla \cdot (\bar{\rho} u k) = \nabla \cdot (\bar{\rho} v_k \nabla k) + P - \varepsilon$$

$$v_k = c_k \Delta k^{1/2} \quad \varepsilon = c_\varepsilon k^{3/2} \Delta^{-1}$$

RNG SGS model

$$\mu_t = \mu \left[ 1 + H \left( \frac{\mu_s^2 \mu_{\text{tot}}}{\mu^3} - C \right) \right]^{1/3}$$

$$\mu_s = \left( 0.157 V^{\frac{1}{3}} \right)^2 \sqrt{2S_{ij}S_{ij}}$$

Dynamic Smagorinsky SGS model

The model is based on a relation between SGS stresses at different scales ( Germano, 1991 )

## Different Energy spectrums

$$E_{11}(k_1) = C_1 \langle \varepsilon \rangle^{2/3} k_1^{-5/3}$$

Kolmogorov -5/3 spectrum – K41 law

$$E_{11}(k_1) = C_1 \varepsilon^{2/3} k_1^{-5/3} (k_1 L)^{-\mu/9}$$

Kolmogorov spectrum – K62 law

where  $\mu$  is the internal intermittency and  $L$  is the integral length scale.

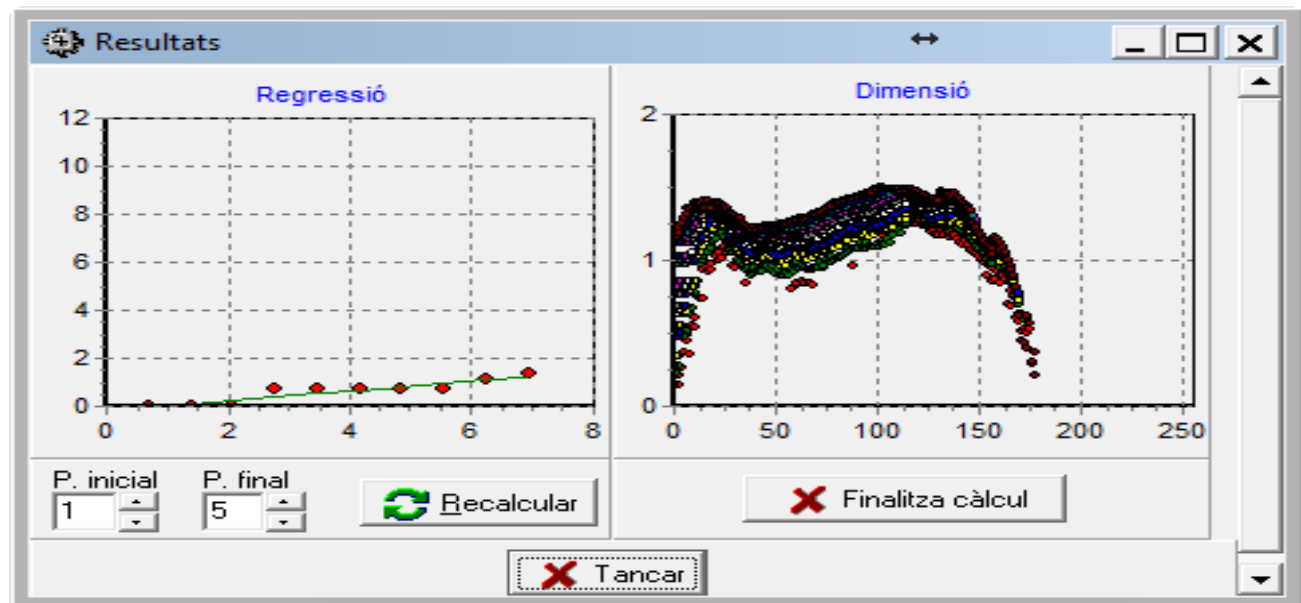
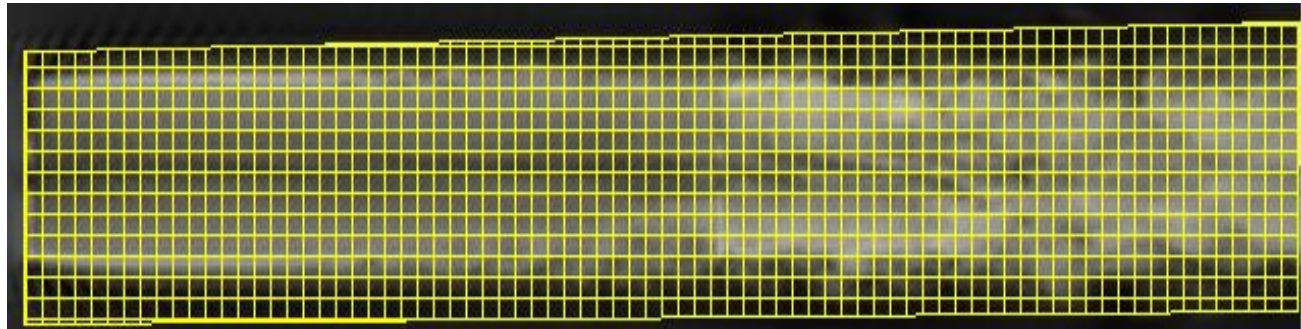
$$E(k) = C \varepsilon^{2/3} k^{-5/3} \left( \frac{kL}{[(kL)^2 + c_L]^{1/2}} \right)^{5/3+p_0} \exp \left\{ -\beta \left\{ [(k\eta)^4 + c_\eta^4]^{1/4} - c_\eta \right\} \right\}$$

The von Karman spectrum is a simple model for the universal form.

The von Karman spectrum can be used to reconstruct the unresolved velocity fields in LES.

$p_0 = 4$ ,  $C = 1.5$ ,  $\beta = 5.2$ ,  $c_L = 6.78$ ,  $c_\eta = 0.4$

# ImaCal: mesh 4 mln. cell



# ImaCal: mesh 4 mln. cell

